Workshop on the
Financial Economics of Insurance
Asset Allocation and Asset Pricing$^{1}$

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Role of institutions in asset pricing

▶ Most capital in financial markets is intermediated via banks, insurance companies, pension funds, mutual funds, and hedge funds.
▶ Frictionless view: Institutions implement heterogeneous beliefs or preferences of investors.
▶ Frictional view: Institutions face leverage, value-at-risk, short-sale constraints, and other agency problems. These frictions show up in asset prices.
Holdings of insurance companies

Source: Chodorow-Reich, Ghent, and Haddad (2016, Figure 2)
Reaching for yield in the corporate bond market

- Required capital depends on the bond rating but there is significant yield variation within each category.

<table>
<thead>
<tr>
<th>NAIC Category</th>
<th>Credit Ratings</th>
<th>Five-Year Cumulative Default Rates (1990–2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exempt</td>
<td></td>
</tr>
<tr>
<td>F  government</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAIC 1 (highest)</td>
<td>AAA, AA, A</td>
<td>Investment grade 0.3% 0.00%, 0.09%, 0.69%</td>
</tr>
<tr>
<td>NAIC 2</td>
<td>BBB</td>
<td>Investment grade 0.96% 2.62%</td>
</tr>
<tr>
<td>N 3</td>
<td>BB</td>
<td>High yield/ speculative grade 3.39% 6.76%</td>
</tr>
<tr>
<td>NAIC 4</td>
<td>B</td>
<td>High yield/ speculative grade 7.38% 8.99%</td>
</tr>
<tr>
<td>N 5</td>
<td>CCC</td>
<td>High yield/ speculative grade 16.96% 34.38%</td>
</tr>
<tr>
<td>NAIC 6 (lowest)</td>
<td>CC or below</td>
<td>High yield/ speculative grade 19.50%</td>
</tr>
</tbody>
</table>

Table II

NAIC Risk-Based Capital Requirements

This table summarizes National Association of Insurance Companies (NAIC) post-tax capital requirement factors (NAIC Risk-Based Capital Newsletter, 10/12/2001). Default rates are from Fitch Ratings Global Corporate Finance 2010 Transition and Default Study.

Source: Becker and Ivashina (2015, *JF*, Table 2)
Corporate bond holdings across NAIC categories

(a) Holdings by NAIC categories

Source: Becker and Ivashina (2015, *JF*, Figure 1a)
Corporate bond holdings within NAIC category 1

(b) Holdings by yields and CDS spreads (NAIC category 1 only)

Source: Becker and Ivashina (2015, *JF*, Figure 1b)
Firesale of corporate bonds following downgrades

- Volume and price dynamics around bond downgrades.
- **Identification problem**: When a bond gets downgraded, risk weights increase but so does risk.
- Ellul, Jotikhastira, and Lundblad (2012, *JFE*) compare bonds held by insurers with high and low RBC ratios.
Firesale of corporate bonds following downgrades

Ellul et al. (2015, JF) present related evidence on ABS downgrades during the financial crisis.
Duration hedging and the slope of demand curves

- Domanski, Shin, and Sushko (2017) consider a model of duration hedging.
- For low enough yields, demand curves become upward-sloping.
Duration hedging and the slope of demand curves

- For low enough yields, demand curves become upward-sloping.
- The prediction finds support in the data using detailed portfolio holdings of German life insurers.
The size of the P&I sectors and long-maturity yield spreads

- More broadly, the size of the pension and insurance sectors is negatively related to the 30y-10y yield spread.

Source: Greenwood and Vissing-Jorgensen (2018, Figure 2)
Value-maximizing insurers and portfolio choice

- Consider an insurer that invests premiums and retained earnings, $A_0$, subject to the RBC constraint to maximize value

$$\max_{x_I} \mathbb{E}[MA_1] - C(K),$$

with $M$ the SDF and subject to the budget constraint,

$$A_1 = A_0(R_1^e x_I + r_f).$$
Portfolio puzzle

- With $\mathbb{E}[\text{MR}^e_1] = 0$ for all assets, the optimization problem simplifies to

  $$\max_{x_t} A_0 - C(K).$$

- Hence, the optimal strategy of the insurer is to minimize $C(K)$.

  $\Rightarrow$ The optimal portfolio is to just hold Treasuries, which is at odds with earlier facts.
Portfolio puzzle: Potential solutions in the literature

- Two types of models have been considered
  2. Insurers generate different returns on the same asset (Chodorow-Reich, Ghent, and Haddad, 2016).
Intermediary asset pricing with insurance companies

Key ingredients:

- Households face mortality risk and optimally purchase annuities, which determines the size of the insurance sector.
- Households and other intermediaries face leverage constraints (Black, 1972, and Frazzini and Pedersen, 2013).
- Insurers maximize firm value subject to the RBC constraint.
- Portfolio choice is determinate as insurance companies relax the economy’s leverage constraints.
Firms

- Dividends of non-insurance ("other") companies have a single-factor structure

\[ D_O = \mu_{DO} + \beta_O F + \epsilon_O, \]

with \( \text{Var}(\epsilon_O) = \sigma^2_{\epsilon} I, \) \( \text{Var}(F) = \sigma^2_F. \)

- A continuum of insurance companies with dividend \( D_I. \)

- Denote \( D = (D'_O, D_I)' \), \( \mu = \mathbb{E}[D], \) \( \Sigma = \text{Var}(D). \)
Insurance companies: Assets

- Insurance companies start with retained earnings $E$ and sell $Q_I$ annuities at price $P_a = \theta \delta r_f^{-1}$, where
  - $\delta$: Survival probability.
  - $\theta \in [1, \delta^{-1}]$: Markup.
  - $r_f$: Gross risk-free rate (exogenous).
- The return on annuities: $r_a = r_f (\theta \delta)^{-1}$.
- Initial assets: $A_{I0} = E + Q_I P_a$. 
Insurance companies: Regulation

- The insurer incurs a cost of regulation, $C(K) = \phi K^{-1}$.
- $K = E/\hat{E}$ is the RBC ratio, where $\hat{E}$ is the required capital.
- $\hat{E} = x_l' D(\rho) x_l$, with $D(\cdot)$ diagonal, where $\rho$ are risk weights.
Insurance companies: Regulation

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- \( K = E/\hat{E} \) is the RBC ratio, where \( \hat{E} \) is the required capital.
- \( \hat{E} = x_DD(\rho)x_l \), with \( D(\cdot) \) diagonal, where \( \rho \) are risk weights.

- The risk weights typically reflect the riskiness of assets, \( \rho = \rho(\beta_0, \eta) \), with \( \rho_{\beta_0} \geq 0 \).
- \( \eta \) measures the capital intensity of the insurers’ liabilities.
- Assume

\[
\rho_{\beta_0\eta} = \frac{\partial^2 \rho}{\partial \beta_0 \partial \eta} > 0,
\]

implying that increases in an asset’s riskiness are more costly for more constrained companies.
Insurance companies: Dividend and objective

- The insurer’s dividend equals

\[ D_I = x'_I D_O + (A_{I0} - x'_I P_O - C(K))r_f \]

\[ = x'_I D_O + \left( E - C(K) + Q_I \frac{\delta}{r_f} (\theta - 1) - x'_I P_O \right) r_f , \]

where \( Q_I \delta (\theta - 1) \) are the underwriting profits.

- If the insurer invests in bonds, \( x_I = 0 \), then

\[ D_I = E_O r_F + Q_I \delta (\theta - 1). \]

- Insurance companies select \( x_I \) to maximize firm value, \( P_I \).
Households: Objective and constraints

- Objective function

\[ E[W_{H1}] - \frac{\gamma_H}{2} \text{Var}[W_{H1}], \]

subject to the budget constraint and leverage constraint

\[ W_{H1} = x_H' D_1 + (W_{H0} - x_H' P_0) r_a, \]
\[ x_H' P_0 \leq W_{H0}. \]
Other institutions also have mean-variance preferences

\[ E[W_{j1}] - \frac{\gamma_j}{2} \text{Var}[W_{j1}], \]

subject to their budget constraint and leverage constraints

\[ W_{j1} = x_j' D_1 + (W_{j0} - x_j' P_0) r_f, \]
\[ x_j' P_0 \leq W_{j0}/m_j, \]

where \( m_j \) captures heterogeneity in leverage constraints.
Market clearing

- The supply of each asset is normalized to 1.
- Market clearing implies

\[ x_H + \sum_j x_j + \hat{x}_I = \iota, \]

where \( \hat{x}_I = (x'_I, 0)' \).
Asset prices: Solution

- Define

\[
\frac{1}{\gamma} = \frac{1}{\gamma_H} + \sum_j \frac{1}{\gamma_j},
\]

\[
r = \frac{\gamma}{\gamma_H} r_a + \sum_j \frac{\gamma}{\gamma_j} r_f,
\]

\[
\lambda = \frac{\gamma}{\gamma_H} \lambda_H + \sum_j \frac{\gamma}{\gamma_j} \lambda_j,
\]

with \(\lambda_i\) the Lagrange multipliers on the leverage constraints.

- Combining first-order conditions and market clearing implies

\[
P = \frac{1}{r + \lambda} (\mu - \gamma \sum \iota + \gamma \sum \hat{X}_i).
\]
Asset prices: Solution

- The prices of other companies, $P_O$, are given by
  \[ \frac{1}{r + \lambda} (\mu_D O - \gamma O O). \]

- The price of insurers, $P_I$, is given by
  \[ \frac{1}{r + \lambda} (\mu I - \gamma O O O I) = \]
  \[ \frac{1}{r + \lambda} (x'_I (\mu_D O - P_O r_f) + (E - C(K)) r_f + Q_I \delta(\theta - 1) - \gamma O O O x_I) \]
Optimal strategy of insurers and implications

- Optimal strategy of the insurer, with $\theta \simeq \delta$ so that $r \simeq r_f$, is

$$x_i^* = \frac{E}{\phi} \frac{\lambda}{r(r + \lambda)} D(\rho)^{-1} \left( \mu_D\nu_O - \gamma \left( \beta_O\beta'_O\nu_O\sigma_F^2 + \sigma^2_{\epsilon}\nu_O \right) \right).$$

- Implication #1

  In a frictionless economy, $\lambda = 0$, it is optimal to hold a portfolio of risk-free bonds, $x_i^* = 0$.

- This is the “portfolio puzzle” highlighted before.
Optimal strategy of insurers and implications

- Optimal strategy of the insurer, with $\theta \simeq \delta$ so that $r \simeq r_f$, is

$$x_i^* = \frac{E}{\phi} \frac{\lambda}{r(r + \lambda)} D(\rho)^{-1} \left( \mu_D \lambda O - \gamma (\beta O \beta'O \lambda O \sigma_F^2 + \sigma^2_{\epsilon} \lambda O) \right).$$

- Implication #2

The Lagrange multiplier is $\lambda(x_i)$. Insurers choose portfolios to relax aggregate leverage constraints, $\lambda(0) > \lambda(x_i^*)$. 
Optimal strategy of insurers and implications

- Optimal strategy of the insurer, with \( \theta \simeq \delta \) so that \( r \simeq r_f \), is

\[
x^*_I = \frac{E}{\phi} \frac{\lambda}{r(r + \lambda)} D(\rho)^{-1} \left( \mu_D \nu_O - \gamma (\beta_O \beta'_O \nu_O \sigma_F^2 + \sigma^2 \epsilon \nu_O) \right).
\]

- Implication #3

- For a given firm \( n \),

\[
x^*_I(n) = \frac{(c_1 - c_2 \beta_O(n))}{\rho(\beta_O(n))},
\]

with \( c_2 > 0 \), implying that insurers tilt to low-beta assets, even without risk-sensitive regulation, \( \rho(\beta_O) = \rho \).

Risk-sensitive regulation amplifies the tilt.
Optimal strategy of insurers and implications

Optimal strategy of the insurer, with $\theta \simeq \delta$ so that $r \simeq r_f$, is

$$x_i^* = \frac{E}{\phi} \frac{\lambda}{r(r + \lambda)} D(\rho)^{-1} \left( \mu_D \lambda O - \gamma \left( \beta_O \beta' O \lambda O \sigma^2_F + \sigma^2 \epsilon O \right) \right).$$

Implication #4

If a firm's beta increases, and the firm is downgraded, insurers sell. As $\rho_{\beta O \eta} > 0$, the selling pressure is larger if a bond is primarily held by constrained companies, potentially leading to firesales.
Equity prices and returns of insurance companies

- The value of an insurance company (beyond retained equity) depends on two terms,

\[
P_I = \frac{1}{r + \lambda} \left( x'_I (\mu_D I - P_O r_f - \gamma \Sigma O I O) - C(K) r_f + Q_I \delta(\theta - 1) x_I + E r_f \right).
\]

- Relaxing leverage constraints
- Market power

- As in Frazzini and Pedersen (2013), the SML is too flat

\[
\mathbb{E}[r_O] = r + \lambda + \beta^r_O \Lambda,
\]

This implies for a stock’s alpha

\[
\alpha_O = \lambda(1 - \beta^r_O).
\]

- The predictions for the alpha and beta of insurers depend on the tightness of the risk regulation and leverage constraints.
Alpha and beta of a portfolio of listed life insurers

▶ Empirically, there is a large shift in the alpha and beta of insurance companies since the financial crisis.

<table>
<thead>
<tr>
<th>Date</th>
<th>Market beta</th>
<th>Alpha (annualized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990m1</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2000m1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>2010m1</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2020m1</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

![Market beta chart](chart1.png)
![Alpha chart](chart2.png)
Summary

- The same model that explains the design and pricing of insurance products as well as reinsurance decisions, can be used to think about the insurers’ asset allocation decision.

- **Broader question:** Quantitative impact of risk regulation on the pricing of fixed income assets and the size of credit markets?

- Koijen and Yogo (forthcoming, *JPE*) develop an equilibrium asset pricing model that is consistent with rich heterogeneity in holdings data that could be useful.