Workshop on the Financial Economics of Insurance
Modeling Demand[1]

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Motivation

- Even if the goal is to study supply, need to estimate demand elasticities to decompose prices into markups (market power) vs. marginal cost (financial frictions).
- Need to model insurance market equilibrium for counterfactuals and welfare analysis.
- Approaches to modeling demand.
  1. Life-cycle model: Cannot explain why demand is different for close substitutes.
  2. Demand systems in product space (e.g., Deaton and Muellbauer 1980).
  3. Demand systems in characteristics space (e.g., Berry, Levinsohn, and Pakes 1995).

Why model demand in characteristics space?

- Insurers offer many policies that are close substitutes.
- These policies are differentiated by contract characteristics (e.g., maturity) and insurer characteristics (e.g., rating).
- Demand systems in characteristics space naturally model products as close substitutes if they have similar characteristics.
- Parsimonious model whose parameters increase in the number of characteristics (not number of products).
- Other applications of random-coefficients logit model in retail financial markets.
  - Health insurance (Einav et al. 2017).
  - Banking (Egan et al. 2017).
  - Mutual funds (Hortaçsu and Syverson 2004, Shin 2014).

Discrete-choice problem

- Investor can choose one of I contracts or an outside asset.
- For an investor with preferences (α, β), the indirect utility from choosing contract i is

$$u_i = \alpha P_i + \beta' \mathbf{x}_i + \xi_i + \epsilon_i$$

  - P_i: fee.
  - $\mathbf{x}_i$: Contract and insurer characteristics.
  - ξ_i: Unobserved (to the econometrician) characteristics.
  - ϵ_i: Logit error drawn from a type 1 extreme value distribution.
- Probability that investor chooses contract i is

$$q_{i,t}(\alpha, \beta) = \frac{\exp\{\alpha P_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t}\}}{1 + \sum_{j=1}^{I} \exp\{\alpha P_{j,t} + \beta' \mathbf{x}_{j,t} + \xi_{j,t}\}}$$

Random-coefficients logit model

- Heterogeneity in demand elasticities across investors, where α is lognormal and β is normal.
- Market share for contract i in period t:

$$Q_{i,t} = \int q_{i,t}(\alpha, \beta)\ dF(\alpha, \beta)$$

- Cross-price elasticity is

$$\frac{\partial \log(Q_{i,t})}{\partial \log(P_{j,t})} = \frac{P_{j,t}}{Q_{i,t}} \int -\alpha q_{i,t}(\alpha, \beta) q_{j,t}(\alpha, \beta)\ dF(\alpha, \beta)$$

- More realistic than the logit model, which implies substitution proportional to market share.

$$\frac{\partial \log(Q_{i,t})}{\partial \log(P_{j,t})} = -\alpha P_{i,t} Q_{j,t}$$

Estimation of the random-coefficients logit model

- Estimation by GMM based on

$$\mathbb{E}[\xi_{i,t}|\mathbf{z}_{i,t}] = 0$$

- $\xi_{i,t}$ is not analytical, so evaluation of the moment function requires a simulation.
- Rewrite the market share as

$$Q(\overline{\alpha}, \sigma_\alpha^2, \sigma_\beta^2, \delta_{i,t}) = \int \frac{\exp\{-e^{-\nu_\alpha} P_{i,t} + \nu_\beta' \mathbf{x}_{i,t} + \delta_{i,t}\}}{1 + \sum_{j=1}^{I} \exp\{-e^{-\nu_\alpha} P_{j,t} + \nu_\beta' \mathbf{x}_{j,t} + \delta_{j,t}\}} \, dF(\nu_\alpha, \nu_\beta)$$

where

$$\begin{aligned} \delta_{i,t} =& \overline{\beta}' \mathbf{x}_{i,t} + \xi_{i,t} \\ \nu_\alpha \sim& \mathbb{N}(\overline{\alpha} - \sigma_\alpha^2/2, \sigma_\alpha^2) \\ \nu_\beta \sim& \mathbb{N}(\mathbf{0}, \mathrm{diag}(\sigma_\beta^2)) \end{aligned}$$

Evaluation of the moment function

1. Start with guess $(\overline{\alpha}, \sigma_\alpha^2, \sigma_\beta^2)$ and $\delta_{i,t}(1)$.
2. Iterate until convergence:

$$\delta_{i,t}(n+1) = \delta_{i,t}(n) + \log(Q_{i,t}) - \log(Q(\mu_\alpha, \sigma_\alpha^2, \sigma_\beta^2, \delta_{i,t}(n)))$$

3. Then $\xi_{i,t} = \delta_{i,t}(N) - \overline{\beta}' \mathbf{x}_{i,t}$.

Application to variable annuities

- Contract characteristics: Fee, rollup rate, number of investment options, and GLWB.
- Insurer characteristics: A.M. Best rating and fixed effects.
- Outside asset: Mutual funds.
- Need two instruments with two endogenous variables (fee and rollup rate).
  1. Reserve valuation: Value of existing liabilities.
  2. Reinsurance share of VA: Constrained insurers use reinsurance.
- Motivation:
  1. Reserves determined by AG43 (30% conditional tail expectation). No reason to believe investors value VA the same way as regulators.
  2. Most of reinsurance is shadow insurance, which investors have little knowledge of beyond what's reflected in ratings.

Estimated model of variable annuity demand

Variable	Mean	Standard deviation
Fee	-3.37	0.30
	(0.13)	(0.05)
Rollup rate	0.18	
	(0.01)	
Investment options	0.11	
	(0.01)	
GLWB	17.02	
	(2.64)	
Share class		
A	-9.01	
	(1.60)	
C	2.01	
	(0.62)	
I	-13.82	
	(2.34)	
L	4.99	
	(1.05)	
O	-5.60	
	(1.03)	
X	3.86	
	(0.82)	
A.M. Best rating	0.73	
	(0.10)	
Observations	32,419	

Decomposition of fees and rolloup rates

- Optimality condition for fee:

$$\log\left(P_{i,t} - \frac{1}{\epsilon_{P,i,t}}\right) = \log(V_{i,t}) + \log(\lambda_{n,t})$$

- Optimality condition for rollup rate:

$$\log\left(\frac{\epsilon_{r,i,t}}{\epsilon_{P,i,t}}\right) = \log\left(\frac{\partial V_{i,t}}{\partial r_{i,t}}\right) + \log(\lambda_{n,t}) + \Omega_{i,t}$$

- Parametric assumptions:
  - Option value:

$$V_{i,t} = \exp\{\delta' \mathbf{y}_{i,t} + \exp\{\Delta' \mathbf{y}_{i,t} + \nu_{i,t}\} r_{i,t} + \eta_{i,t}\}$$

  - Shadow cost of capital:

$$\lambda_{n,t} = \exp\{\Gamma' \mathbf{z}_{n,t} + \gamma_n\}$$

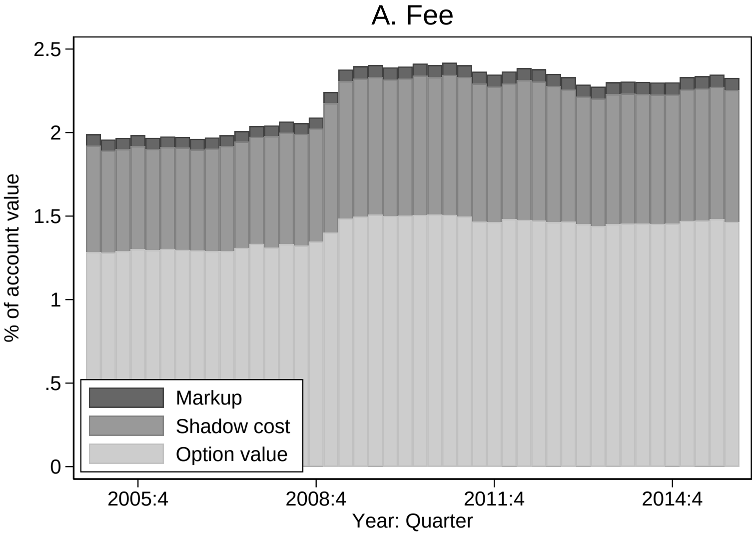
  - Constraint on rollup rate:

$$\Omega_{i,t} = \omega \mathbb{1}_{\{r_{i,t} \neq -1\}}$$

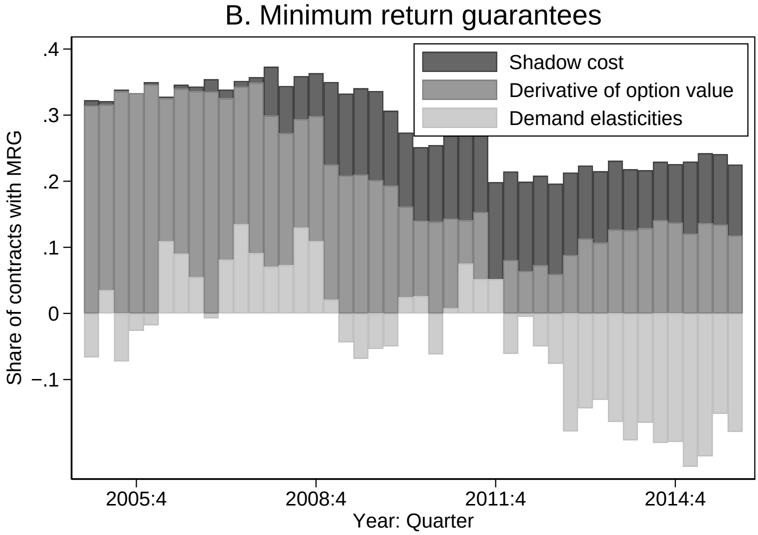
Estimated model of variable annuity supply

Variable	Coefficient	Standard error
Panel A: Level of option value		
Investment options	0.19	(0.02)
GLWB	49.93	(0.28)
Share class		
A	-42.64	(0.96)
C	10.41	(0.39)
I	-81.65	(1.32)
L	18.93	(0.29)
O	-20.31	(0.98)
X	15.59	(0.32)
Panel B: Slope of option value		
Investment options	-0.01	(0.01)
GLWB	-12.29	(0.31)
Share class		
A	44.32	(1.00)
C	-8.86	(0.36)
I	77.34	(1.29)
L	-15.11	(0.20)
O	21.08	(1.08)
X	-14.15	(0.21)
Panel C: Shadow cost		
A.M. Best rating	-3.10	(0.26)
Reserve valuation	1.16	(0.24)
Variable annuities reinsured	1.44	(0.20)
Panel D: Constraint on rollup rate		

Decomposition of fees



Decomposition of rollup rates



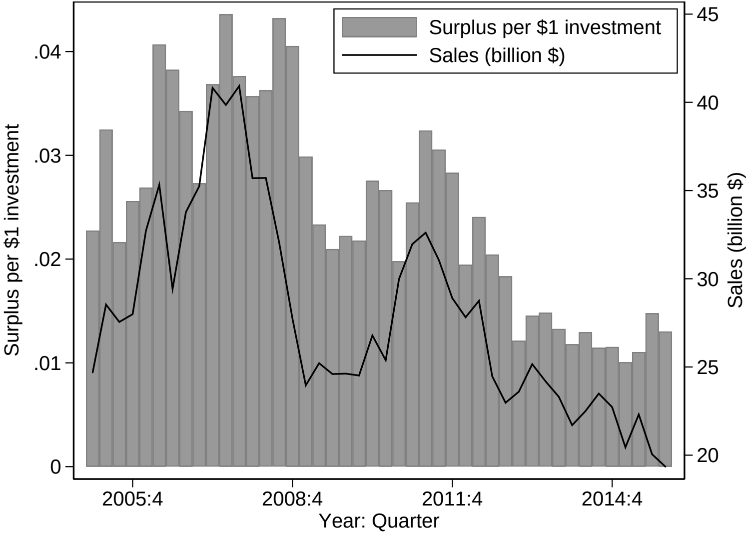
Consumer surplus

- What is the total consumer surplus from variable annuities?
- Log-sum formula (Small and Rosen 1981):

$$\int \frac{\log(1 + \sum_{i=1}^{I} \exp\{\alpha P_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t}\})}{-\alpha} \, dF(\alpha, \beta)$$

- Analogous to asking what is the value of defined-benefit plans, relative to an alternative in which all plans are defined-contribution.
- Possible because we observe both prices and quantities for variable annuities.

Variable annuity fees and marginal cost



Summary

- An equilibrium framework to study insurance markets.
  - Supply: Choose prices and contract characteristics to maximize firm value subject to risk-based capital cost.
  - Demand: Random-coefficients logit model.
  - Demand estimation.
  - Use the model for policy counterfactuals and welfare analysis.
- Next application: Reinsurance.