Workshop on the
Financial Economics of Insurance
Modeling Supply

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Composition of life insurer liabilities in 2015

<table>
<thead>
<tr>
<th>Liability</th>
<th>Trillion $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable annuities (separate accounts)</td>
<td>1.8</td>
</tr>
<tr>
<td>Life insurance</td>
<td>1.5</td>
</tr>
<tr>
<td>Traditional annuities</td>
<td>1.0</td>
</tr>
<tr>
<td>Pension fund liabilities</td>
<td>0.7</td>
</tr>
<tr>
<td>Other reserves (accident &amp; health)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Risk-sharing functions of life insurers:

1. **Diversify idiosyncratic risk**: Traditional life/health products.
2. **Market risk insurance**: Reallocate aggregate risk across
   - Investors with heterogeneous risk preferences (Dumas 1989).
   - Generations, taking on the traditional role of pension plans and Social Security (Allen and Gale 1997).
Questions

1. What do life insurers insure?
   - Market risk through minimum return guarantees.
   - **Variable annuity = Mutual fund + Long-dated put option**
   - $1.5 trillion or 35% of U.S. life insurer liabilities in 2015.

2. How do they insure market risk?
   - Financial frictions and market power interact with aggregate shocks.
   - After the 2008 financial crisis,
     - Pricing: Fees increased and sales decreased.
     - **Contract characteristics:** Insurers made guarantees less generous or exited to limit risk exposure.
     - Moved liabilities off balance sheet through reinsurance.
   - Never fully recovered because of the low interest rate environment.
     - Negative duration and negative convexity.
     - During 2010–2017, insurance stocks have 10-year Treasury beta of $-1.28$ ($0.16$).
Example: MetLife Series VA

- Sold by MetLife Insurance Company USA.
- American Funds Growth Allocation Portfolio: Mutual fund with a target equity allocation of 70–85%.
- Annual base contract expense of 1.3%.
- Guaranteed Lifetime Withdrawal Benefit: Optional minimum return guarantee with
  - Annual fee of 0.5%.
  - Rollup rate (guaranteed return) of 5%.
  - Withdrawal rate of 5%.
Example of a guaranteed living withdrawal benefit

![Graph showing data over time with labels for withdrawal at 5% and account value]
Example of a guaranteed living withdrawal benefit
Example of a guaranteed living withdrawal benefit

<table>
<thead>
<tr>
<th>Date</th>
<th>Withdrawal at 5%</th>
<th>Account value</th>
<th>Rollup rate of 5%</th>
<th>Guaranteed amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016:4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
How are variable annuities regulated?

- Insurance regulators and rating agencies use the risk-based capital ratio.

\[ \text{RBC} = \frac{\text{Assets} - \text{Reserves}}{\text{Required capital}} \]

- Value of minimum return guarantee (put option) increases with lower stock prices, lower interest rates, and higher volatility.
  1. Reserves increase because of revaluation.
  2. Required capital increases through risk exposure.

- Accounting value of minimum return guarantees:

\[ \text{Reserve valuation} = \frac{\text{Reserves}}{\text{Account value}} \]
Value-at-risk constraint

- The constraint that insurers faces may not be just regulatory, but also economic.

- Value-at-risk constraint:

\[
\Pr \left( \frac{\text{Liabilities}}{\text{Assets}} \epsilon \leq 1 \right) = F \left( \frac{\text{Assets}}{\text{Liabilities}} \right) \geq \kappa
\]

- Rewrite as

\[
\frac{\text{Assets} - \text{Liabilities}}{\phi \text{Liabilities}} \geq 1
\]

where \( \phi = F^{-1}(\kappa) - 1 \).

- Insurer with more conservative risk management chooses higher \( \kappa \) or lower variance.
Why do insurers not hedge variable annuity risk?

1. Insurers may not want to hedge.
   ▶ Risk shifting under limited liability and guaranty funds.
   ▶ Hedge positions differ depending on whether insurer targets economic, statutory, or GAAP capital.

2. Insurers cannot hedge perfectly because of model uncertainty.

3. Limited supply of long-dated puts.
   ▶ Someone has to hold aggregate risk, and insurers may have comparative advantage over other institutions.
### Risk exposure of U.S. life insurers

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock market</td>
<td>1.36</td>
<td>0.56</td>
<td>2.56</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.15)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>10-year Treasury</td>
<td>-0.01</td>
<td>-0.38</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.29)</td>
<td>(0.66)</td>
</tr>
<tr>
<td>Alpha (%)</td>
<td>-0.22</td>
<td>0.35</td>
<td>-1.14</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.47)</td>
<td>(1.70)</td>
</tr>
<tr>
<td>Observations</td>
<td>228</td>
<td>108</td>
<td>24</td>
</tr>
</tbody>
</table>
Data on the variable annuity market

  - Quarterly contract-level data on sales, fees, and characteristics.

  - Annual data on VA account value, reserve value, and amount of reserves reinsured.

  - Annual financial statements.
  - A.M. Best rating.
A summary of the variable annuity market

<table>
<thead>
<tr>
<th>Year</th>
<th>Billion $</th>
<th>% of total liabilities</th>
<th>Number of insurers</th>
<th>Reserve valuation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>1,071</td>
<td>35</td>
<td>45</td>
<td>0.9</td>
</tr>
<tr>
<td>2006</td>
<td>1,276</td>
<td>38</td>
<td>47</td>
<td>0.8</td>
</tr>
<tr>
<td>2007</td>
<td>1,435</td>
<td>41</td>
<td>46</td>
<td>0.8</td>
</tr>
<tr>
<td>2008</td>
<td>1,068</td>
<td>34</td>
<td>44</td>
<td>4.1</td>
</tr>
<tr>
<td>2009</td>
<td>1,195</td>
<td>35</td>
<td>43</td>
<td>3.4</td>
</tr>
<tr>
<td>2010</td>
<td>1,344</td>
<td>36</td>
<td>43</td>
<td>2.5</td>
</tr>
<tr>
<td>2011</td>
<td>1,358</td>
<td>35</td>
<td>42</td>
<td>4.9</td>
</tr>
<tr>
<td>2012</td>
<td>1,434</td>
<td>36</td>
<td>39</td>
<td>3.9</td>
</tr>
<tr>
<td>2013</td>
<td>1,606</td>
<td>37</td>
<td>40</td>
<td>1.8</td>
</tr>
<tr>
<td>2014</td>
<td>1,599</td>
<td>37</td>
<td>38</td>
<td>2.3</td>
</tr>
<tr>
<td>2015</td>
<td>1,499</td>
<td>35</td>
<td>38</td>
<td>2.9</td>
</tr>
</tbody>
</table>
Variable annuity sales

![Graph showing variable annuity sales and mutual fund sales over time]

- Variable annuity sales (billion $)
- Mutual fund sales (billion $)

Year: Quarter

Number of insurers with minimum return guarantees
Fees on minimum return guarantees

A. Fee

- Total fee
- Fee on MRG

% of account value

Year: Quarter

Rollup rates

B. Rollup rate

The graph shows the rollup rates and the share of contracts with MRG from 1999:4 to 2014:4. The rollup rate is represented by a black line, which peaks around 2011:4. The share of contracts with MRG is represented by a grey area, indicating a decreasing trend over the years.
Evidence on supply-side fragility

- Fees increased and sales decreased.
- **Changing contract characteristics**: Insurers made guarantees less generous to limit risk exposure.
  - Lower rollup rates (intensive margin).
  - Exit the market for guarantees (extensive margin).
- **Hypothesis**: Higher valuation of existing liabilities lowers risk-based capital.
- In the cross section, insurers with larger increases in reserve valuation
  1. Reduce sales more.
Sales growth vs. change in reserve valuation

A. Sales growth

The graph shows the relationship between sales growth and change in reserve valuation for various insurance companies. The companies are marked with different symbols, and the line indicates a negative correlation between the two variables.

Offered GLWB in 2007:
- Yes
- No
Change in variable annuity reinsurance vs. change in reserve valuation

B. Variable annuity reinsurance

Offered GLWB in 2007:
- Yes
- No

Change in reinsurance share (%) vs. change in reserve valuation
A model of variable annuity supply

▶ Key frictions:


▶ In response to an adverse shock to risk-based capital, insurer

1. Increases prices.
2. Changes contract characteristics (or exits entirely) to limit risk exposure.
Variable annuity market

- Financial market:
  - Asset price $S_t$ evolves exogenously.
  - SDF that prices all assets: $S_t = \mathbb{E}_t[M_{t,t+1}S_{t+1}]$.

- VA issued in period $t$ matures in period $t + 2$.
  - Fee $P_t$ (per $1$ account value).
  - Rollup rate $r_t$.
  - Payoff of minimum return guarantee:
    \[ X_{t,t+2} = \max \left\{ (1 + r_t)^2 - \frac{S_{t+2}}{S_t}, 0 \right\} \]

- Option value: $V_{t,t} = \mathbb{E}_t[M_{t,t+2}X_{t,t+2}]$.

- VA is a mutual fund when $r_t = -1$ (i.e., insurer exits the market for guarantees).
Risk-based capital

- Risk-based capital:

\[ K_t = A_t - L_t - \phi_t L_t \]

- Risk weight \( \phi_t > 0 \) on liabilities.

- Cost of a rating downgrade or regulatory action:

\[ C_t = C(K_t) \]

where \( C' < 0 \) and \( C'' > 0 \).

- Define shadow cost of capital as

\[ \lambda_t = \frac{1 - C'_t(1 + \phi_t)}{1 - C'_t} > 1 \]
Shocks to risk-based capital

- Adverse shock at $t$ (i.e., lower stock prices, lower interest rates, or higher volatility).
  1. Value of existing liabilities $V_{t-1,t}$ rises, lowering risk-based capital.
  2. Marginal cost $V_{t,t}$ increases on new contracts.
- Shadow cost of capital $\lambda_t$ increases through $V_{t-1,t}$.
Insurer’s maximization problem

- Demand: \( Q_t = Q_t(P_t, r_t) \).
  - Decreasing in price: \( \frac{\partial Q_t}{\partial P_t} < 0 \).
  - Increasing in the rollup rate: \( \frac{\partial Q_t}{\partial r_t} > 0 \).
- Insurer cannot offer negative rollup rates: \( r_t \in \{-1\} \cup [0, \infty) \).
- Insurer maximizes firm value:
  \[
  \max_{P_t, r_t} (P_t - V_{t,t}) Q_t - C_t.
  \]
Result 1: Optimal pricing

Optimal fee is

\[ P_t = \frac{1}{\epsilon_{P,t}} + \lambda_t V_{t,t} \]

### Optimal fee is

1. Decreases in semi-elasticity \( \epsilon_{P,t} \).
2. Increases in option value \( V_{t,t} \).
3. Increases in shadow cost of capital \( \lambda_t \).

### Adverse shock increases fee through

1. Higher \( V_{t,t} \).
2. Higher \( \lambda_t \) through revaluation of existing liabilities.
Result 2: Optimal rollup rate

- At an interior optimum, rollup rate satisfies

\[
\frac{\epsilon_{r,t}}{\epsilon_{P,t}} = \lambda_t \frac{\partial V_{t,t}}{\partial r_t}
\]

Otherwise, corner solution \( r_t \in \{-1, 0\} \).

- When shadow cost of capital \( \lambda_t \) is high, insurer
  - Reduces the rollup rate.
  - Can exit the market for guarantees (i.e., \( r_t = -1 \)).

- **Key insight**: Contract characteristics respond to risk-based capital and can lead to market incompleteness.
## Supply-side theory of insurance

<table>
<thead>
<tr>
<th></th>
<th>Traditional view</th>
<th>Supply-side view</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Products</strong></td>
<td>Life/health insurance &amp; traditional annuities</td>
<td>Guaranteed return products</td>
</tr>
<tr>
<td><strong>Insures</strong></td>
<td>Idiosyncratic risk across states</td>
<td>Market risk across investors</td>
</tr>
<tr>
<td><strong>Frictions</strong></td>
<td>Informational</td>
<td>Financial/regulatory &amp; market power</td>
</tr>
<tr>
<td><strong>Consequences</strong></td>
<td>Variation in prices, contract characteristics &amp; degree of market incompleteness</td>
<td></td>
</tr>
</tbody>
</table>
Broader issues

- Mutual funds traditionally pure pass-through institutions with no risk mismatch.
- Growing part of the mutual fund sector sold through life insurers is subject to risk mismatch.
- Similar problem to persistent under-funding of pension funds, but with additional market discipline for publicly traded companies.
- Guaranteed return products are a significant share of life insurer liabilities in Austria, Denmark, France, Germany, Netherlands, and Sweden (ESRB 2015).
- A potential issue to monitor for financial stability.