Financial Economics of Insurance
Portfolio Choice and Asset Pricing[1]

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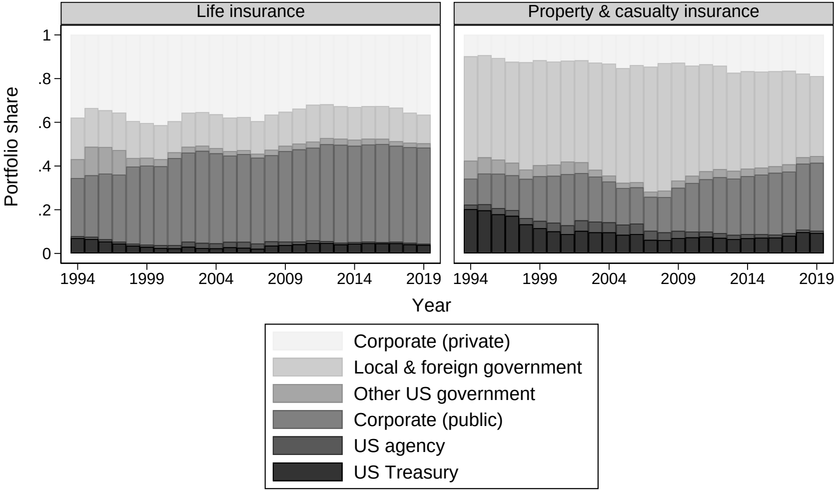
Intermediary asset pricing

- Most financial assets (Treasuries, corporate bonds, and stocks) are held through banks, insurers, pension funds, mutual funds, and hedge funds.
- Frictionless view: Institutional investors simply implement the heterogeneous preferences or beliefs of households.
  - Mutual funds and ETF.
- Intermediary asset pricing.
  - Institutional investors face short-sale, leverage, risk-based capital (RBC), or value-at-risk (VAR) constraints.
  - Their incentives may not align with households because of agency problems (e.g., risk shifting).

Important role of insurers

- Households are willing to pay a convenience yield to insure idiosyncratic risk.
- Cheap access to leverage through insurance underwriting.
- Comparative advantage relative to other institutional investors that are leverage constrained.
- Insurers hold a leveraged portfolio of low-beta assets (Black 1972, Frazzini and Pedersen 2013).

Portfolio composition



NAIC designation

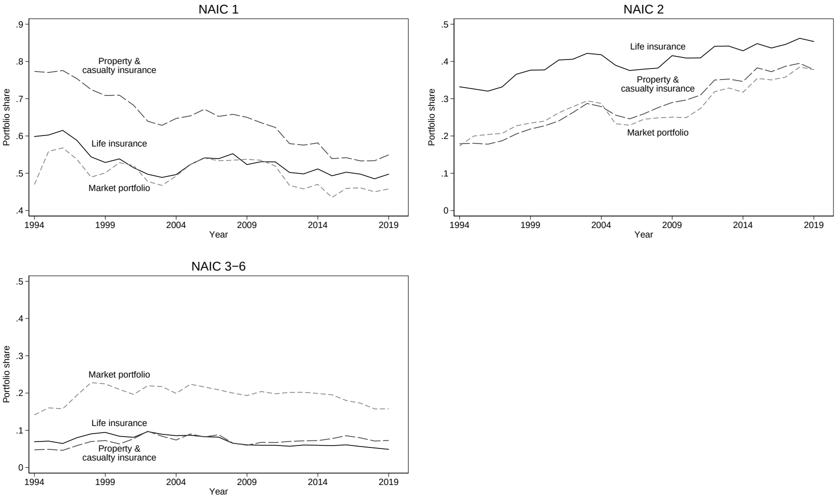
Table II
NAIC Risk-Based Capital Requirements

This table summarizes National Association of Insurance Companies (NAIC) post-tax capital requirement factors (NAIC Risk-Based Capital Newsletter, 10/12/2001). Default rates are from Fitch Ratings Global Corporate Finance 2010 Transition and Default Study.

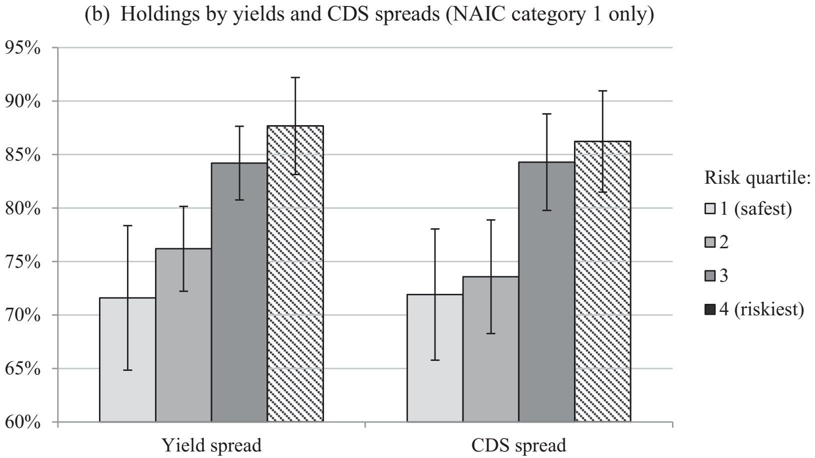
NAIC Category	Credit Ratings		Capital Charge	Five-Year Cumulative Default Rates (1990–2010)
Federal government			Exempt	
NAIC 1 (highest)	AAA, AA, A	Investment grade	0.3%	0.00%, 0.09%, 0.69%
NAIC 2	BBB	Investment grade	0.96%	2.62%
NAIC 3	BB	High yield/ speculative grade	3.39%	6.76%
NAIC 4	B	High yield/ speculative grade	7.38%	8.99%
NAIC 5	CCC	High yield/ speculative grade	16.96%	34.38%
NAIC 6 (lowest)	CC or below	High yield/ speculative grade	19.50%	

Source: Becker and Ivashina (2015, *JF*, Table 2)

Bond portfolio composition

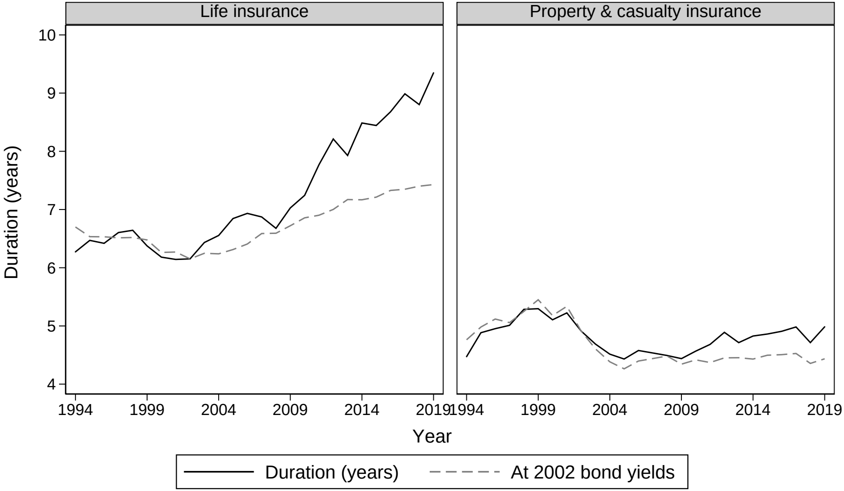


Corporate bond holdings within NAIC category 1

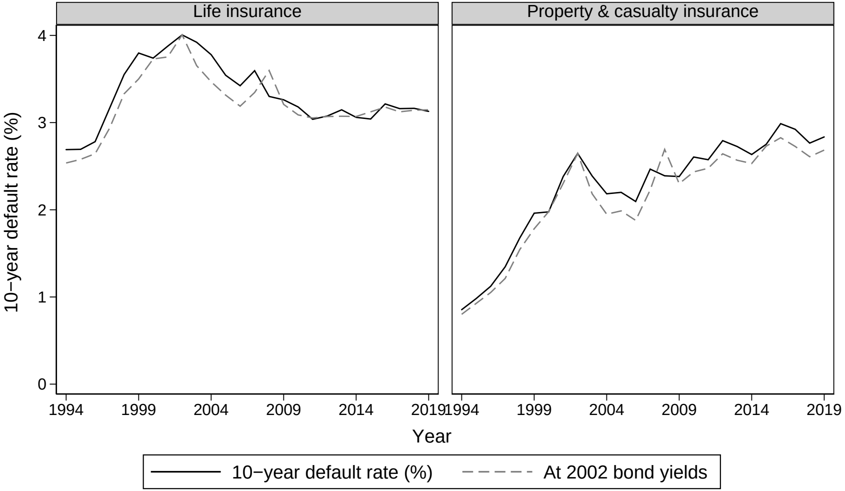


Source: Becker and Ivashina (2015, *JF*, Figure 1b)

Average duration of bond portfolios



Credit risk of bond portfolios



Summary of the portfolio facts

1. Insurers hold more corporate bonds than Treasuries. This portfolio tilt has strengthened over time.
2. Within corporate bonds,
   - Life insurers tilt toward NAIC 2 bonds.
   - Property-casualty insurers tilt toward NAIC 1 bonds.
3. Life insurers leave a duration gap because corporate bonds have a shorter maturity distribution than Treasuries.

- Does theory predict that insurers hold corporate bonds?

Insurer's portfolio choice problem

- Insurer maximizes

$$\max_{\mathbf{x}_I} \mathbb{E}[MA_1] - C(\mathbf{x}_I)$$

where M the SDF.

- Intertemporal budget constraint:

$$A_1 = (R_f + (\mathbf{R} - R_f\mathbf{1})'\mathbf{x}_I)A_0$$

- RBC regulation: $C(\mathbf{x}_I)$ is increasing in $\mathbf{x}_I$ (i.e., riskiness of the portfolio).

A portfolio puzzle for insurers

- Absence of arbitrage implies that

$$\mathbb{E}[M(R_f + (\mathbf{R} - R_f \mathbf{1})' \mathbf{x}_I)] = 1$$

for any $\mathbf{x}_I$.

- Insurer's portfolio choice simplifies to

$$\max_{\mathbf{x}_I} A_0 - C(\mathbf{x}_I)$$

- Minimize $C(\mathbf{x}_I)$ by holding only the riskless asset.

Potential resolutions of the portfolio puzzle

1. Presence of arbitrage allows insurers to earn alpha on corporate bonds and MBS (Chodorow-Reich et al. 2021, Knox and Sørensen 2020).
2. Insurers maximize $\mathbb{E}[A_1]$ instead of $\mathbb{E}[MA_1]$ subject to the RBC constraint (Adrian and Shin 2014, Coimbra and Rey 2017, Ellul et al. 2022).
3. Insurers have relatively cheap access to leverage, and other institutional investors are leverage constrained.

Asset pricing with an insurance sector

- Households and other institutional investors are leverage constrained (Black 1972, Frazzini and Pedersen 2013). They choose a portfolio of risky assets and a riskless asset.
- Households without bequest motives face longevity risk and purchase annuities.
- Insurers choose a portfolio of risky assets and a riskless asset subject to an RBC constraint.
- In equilibrium, insurers hold a leveraged portfolio of low-beta assets, relaxing the leverage constraints of other investors.

Financial assets

- Riskless asset with gross interest R_f.
- Annuities with gross return of 0 conditional on death and

$$R_L = \frac{1}{P_L} = \left(1 - \frac{1}{\epsilon}\right) \frac{R_f}{\pi} \tag{1}$$

conditional on survival.

  - Household do not have a bequest motive and survive with probability π.
  - Insurers have market power and face demand elasticity $\epsilon > 1$.

Risky assets

- Insurers pay out dividends d_I in period 1.
  - Endogenously determined by portfolio choice in period 0.
- Other firms pay out exogenous dividends in period 1, which have a factor structure:

$$\mathbf{d} = \mathbb{E}[\mathbf{d}] + \boldsymbol{\beta} F + \boldsymbol{\nu}$$

  - $\mathbb{E}(F) = 0$ and $\mathrm{Var}(F) = \sigma_F^2$.
  - $\mathbb{E}[\boldsymbol{\nu}] = \mathbf{0}$ and $\mathrm{Var}(\boldsymbol{\nu}) = \mathrm{diag}(\boldsymbol{\sigma}_\nu^2)$.
- Denote $\mathbf{D} = (\mathbf{d}', d_I)'$, $\boldsymbol{\mu} = \mathbb{E}[\mathbf{D}]$, and $\boldsymbol{\Sigma} = \mathrm{Var}(\mathbf{D})$.
- Asset prices $\mathbf{P} = (\mathbf{p}', p_I)'$ in period 0.

Insurers

- Insurers have initial equity E and sell Q units of annuities to households at the price P_L.
- Insurers allocate their assets to $\mathbf{X}_I = (\mathbf{x}_I', 0)'$ units of risky assets in period 0.
  - Insurers cannot hold other insurers.
- RBC:

$$C(\mathbf{x}_I) = \frac{\mathbf{x}_I' \exp(\phi \boldsymbol{\beta}) \mathbf{x}_I}{2}$$

  - Insurers with riskier liabilities have higher $\phi \geq 0$.
- Assets in period 0:

$$A_{I,0} = E - C(\mathbf{x}_I) + P_L Q$$

Insurers' dividends

- Dividends in period 1 are the portfolio value minus annuity claims:

$$
\begin{aligned}
d_I =& \mathbf{d}'\mathbf{x}_I + R_f(A_{I,0} - \mathbf{p}'\mathbf{x}_I) - \pi Q \\
=& \mathbf{d}'\mathbf{x}_I + R_f(E - C(\mathbf{x}_I) - \mathbf{p}'\mathbf{x}_I) + (R_f P_L - \pi)Q
\end{aligned}
$$

- Insurers choose P_L to maximize firm value in period 0.
- Substituting equation (1),

$$
d_I = \mathbf{d}'\mathbf{x}_I + R_f(E - C(\mathbf{x}_I) - \mathbf{p}'\mathbf{x}_I) + \frac{\pi Q}{\epsilon - 1} \qquad (2)
$$

  - If insurers were to hold only the riskless asset,

$$
d_I = R_f E + \frac{\pi Q}{\epsilon - 1}
$$

Households

- Mean-variance preferences conditional on survival:

$$\mathbb{E}[A_{H,1}] - \frac{\gamma_H}{2}\mathrm{Var}(A_{H,1})$$

- Intertemporal budget constraint:

$$A_{H,1} = \mathbf{D}'\mathbf{X}_H + Q$$

- Leverage constraint:

$$\mathbf{P}'\mathbf{X}_H \leq A_{H,0}$$

    - Cannot short annuities.

Institutional investors

- Institutional investor j has mean-variance preferences:

$$\mathbb{E}[A_{j,1}] - \frac{\gamma_j}{2}\mathrm{Var}(A_{j,1})$$

- Intertemporal budget constraint:

$$A_{j,1} = \mathbf{D}'\mathbf{X}_j + R_f(A_{j,0} - \mathbf{P}'\mathbf{X}_j)$$

- Leverage constraint:

$$\mathbf{P}'\mathbf{X}_j \leq \frac{A_{j,0}}{\omega_j}$$

    - $\omega_j > 0$ captures heterogeneity in leverage constraints.

Solving the model

1. Solve portfolio choice for households and other institutional investors.
2. Market clearing to solve for asset prices conditional on the insurers' portfolio.
3. Solve insurers' portfolio choice that maximizes firm value.

Optimal portfolio choice

- Households:

$$\begin{aligned}\mathcal{L}_H =& \mathbb{E}[A_{H,1}] - \frac{\gamma_H}{2}\mathrm{Var}(A_{H,1}) + \lambda_H(A_{H,0} - \mathbf{P}'\mathbf{X}_H) \\ =& \boldsymbol{\mu}'\mathbf{X}_H + (R_L + \lambda_H)(A_{H,0} - \mathbf{P}'\mathbf{X}_H) - \frac{\gamma_H}{2}\mathbf{X}_H'\boldsymbol{\Sigma}\mathbf{X}_H\end{aligned}$$

  - Optimal portfolio:

$$\mathbf{X}_H = \frac{1}{\gamma_H}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - (R_L + \lambda_H)\mathbf{P})$$

- Institutional investors:

$$\begin{aligned}\mathcal{L}_j =& \mathbb{E}[A_{j,1}] - \frac{\gamma_j}{2}\mathrm{Var}(A_{j,1}) + \lambda_j(A_{j,0} - \mathbf{P}'\mathbf{X}_j) \\ =& \boldsymbol{\mu}'\mathbf{X}_j + (R_f + \lambda_j)\left(\frac{A_{j,0}}{\omega_j} - \mathbf{P}'\mathbf{X}_j\right) - \frac{\gamma_j}{2}\mathbf{X}_j'\boldsymbol{\Sigma}\mathbf{X}_j\end{aligned}$$

  - Optimal portfolio:

$$\mathbf{X}_j = \frac{1}{\gamma_j}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - (R_f + \lambda_j)\mathbf{P})$$

Market clearing

- Normalize supply of risky assets to 1 unit each.
- Market clearing:

$$\mathbf{X}_I + \mathbf{X}_H + \sum_{j=1}^{J} \mathbf{X}_j = \mathbf{1}$$

Asset prices

- Asset prices conditional on insurers' portfolio:

$$\mathbf{P}(\mathbf{X}_I) = \frac{1}{R+\lambda}(\boldsymbol{\mu} - \gamma \boldsymbol{\Sigma}(\mathbf{1} - \mathbf{X}_I))$$

where

$$\begin{aligned}
\frac{1}{\gamma} =& \frac{1}{\gamma_H} + \sum_{j=1}^{J} \frac{1}{\gamma_j} \\
R =& \frac{\gamma}{\gamma_H} R_L + \sum_{j=1}^{J} \frac{\gamma}{\gamma_j} R_f \\
\lambda =& \frac{\gamma}{\gamma_H} \lambda_H + \sum_{j=1}^{J} \frac{\gamma}{\gamma_j} \lambda_j
\end{aligned}$$

Asset prices

- Asset prices of other firms:

$$\mathbf{p} = \frac{1}{R+\lambda}(\mathbb{E}[\mathbf{d}] - \gamma \mathrm{Var}(\mathbf{d})\mathbf{1})$$

- Insurers' equity price:

$$p_I = \frac{1}{R+\lambda}(\mathbb{E}[d_I] - \gamma \mathbf{1}' \mathrm{Var}(\mathbf{d})\mathbf{x}_I)$$

Insurers' equity price

- Substituting equation (2),

$$p_I = \frac{1}{R+\lambda}\left(\underbrace{\frac{R-R_f+\lambda}{R+\lambda}(\mathbb{E}[\mathbf{d}]-\gamma\mathrm{Var}(\mathbf{d})\mathbf{1})'\mathbf{x}_I}_{\text{portfolio choice}} + \underbrace{R_f(E-C(\mathbf{x}_I))}_{\text{regulatory frictions}} + \underbrace{\frac{\pi Q}{\epsilon-1}}_{\text{underwriting profits}}\right)$$

Insurers' equity price

1. Portfolio choice.
   - Firm value increases in λ: Other investors can relax leverage constraints by holding risky assets through the insurance sector.
   - Firm value increases in $R - R_f$: Households pay a convenience yield to insure longevity risk. Insurers have a lower funding rate R_f than the effective riskless rate R used for firm valuation.
2. Regulatory frictions due to RBC constraint.
3. Underwriting profits from market power.

Insurers' optimal portfolio

- Insurers' optimal portfolio is

$$\mathbf{x}_I = \frac{R - R_f + \lambda}{R_f(R + \lambda)} \exp(-\phi\boldsymbol{\beta})(\mathbb{E}[\mathbf{d}] - \gamma\sigma_F^2\boldsymbol{\beta}\boldsymbol{\beta}'\mathbf{1} - \gamma\boldsymbol{\sigma}_\nu^2)$$

- Implication 1: If leverage constraints are not binding ($\lambda = 0$) and households do not pay a convenience yield ($R - R_f = 0$), $\mathbf{x}_I = \mathbf{0}$.
- This is the "portfolio puzzle".

Demand for low-beta assets

- Optimal allocation to a risky asset is decreasing in its beta:

$$\frac{\partial x_I(n)}{\partial \beta(n)} = -\frac{R - R_f + \lambda}{R_f(R+\lambda)} \exp(-\phi\beta(n)) \\ \times \left(\phi \mathbf{1}_n' (\mathbb{E}[\mathbf{d}] - \gamma \mathrm{Var}(\mathbf{d})\mathbf{1}) + \gamma \sigma_F^2 (\beta(n) + \boldsymbol{\beta}'\mathbf{1})\right) < 0$$

- Implication 2: Insurers tilt their portfolio toward low-beta assets.
- By doing so, they relax the leverage constraints of other investors.

Sensitivity to risk-based capital

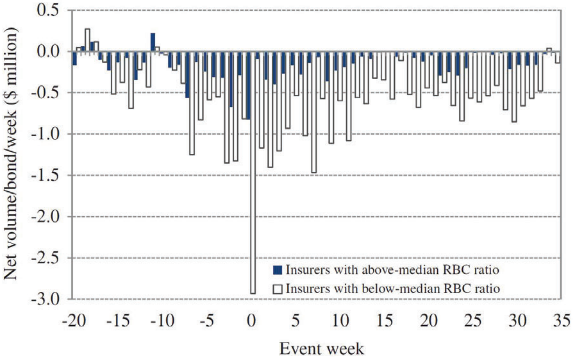
- Sensitivity of portfolio tilt to RBC:

$$\frac{\partial^2 x_I(n)}{\partial\beta(n)\partial\phi} = -\frac{R - R_f + \lambda}{R_f(R+\lambda)} \exp(-\phi\beta(n)) \\ \times [(1-\phi\beta(n))\mathbf{1}_n'(\mathbb{E}[\mathbf{d}] - \gamma \mathrm{Var}(\mathbf{d})\mathbf{1}) \\ - \gamma\sigma_F^2\beta(n)(\beta(n) + \boldsymbol{\beta}'\mathbf{1})] < 0$$

  for sufficiently low $\beta(n)$.
- Implication 3: Insurers' portfolios are more sensitive to beta when RBC is lower (i.e., ϕ is higher).

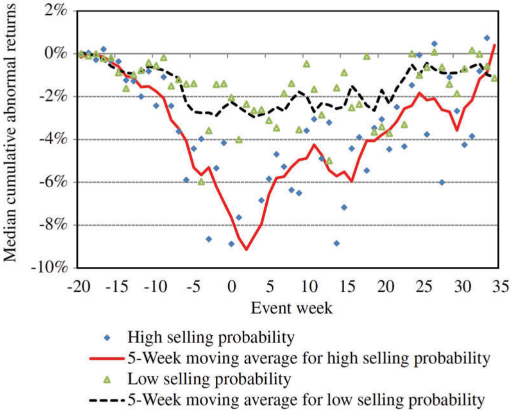
Firesale of downgraded corporate bonds

- Identification problem: When a bond is downgraded, risk weights increase but so does risk.
- Ellul et al. (2012) compare bonds held by insurers with high and low RBC ratios.



Source: Ellul et al. (2012, *JFE*, Figure 1)

Firesale of downgraded corporate bonds



Source: Ellul et al. (2012, *JFE*, Figure 2)

- Ellul et al. (2015) find similar evidence for downgraded ABS during the global financial crisis.

Evidence for implication 3

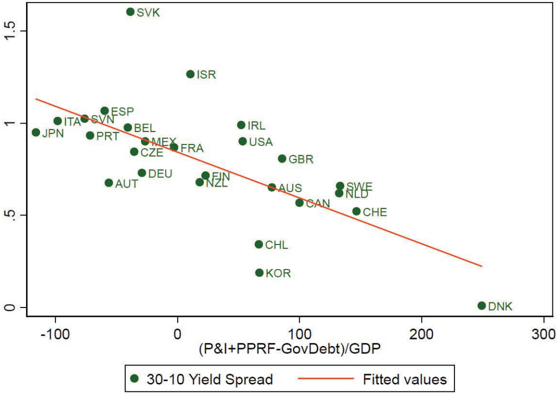
- Becker et al. (2021) find that insurers held on to downgraded non-agency MBS even though they sold downgraded bonds in the rest of their portfolio after the global financial crisis.
  - RBC eliminated for non-agency MBS, which effectively lowers ϕ.
- Ge and Weisbach (2020) find that property-casualty insurers shift toward safer bonds in response to weather shocks.
  - Operating losses tighten RBC or VAR constraints, equivalent to an increase in ϕ.
- Ellul et al. (2022) find that variable annuity insurers reduced equity risk after the global financial crisis.
  - Variable annuity losses tighten RBC constraints, equivalent to an increase in ϕ.

Extensions

- Interest risk mismatch.
  - Shifting from Treasuries to corporate bonds could reduce the duration of the portfolio, worsening the interest risk mismatch.
- Capital structure (Baker and Wurgler 2015, Baker et al. 2020).
  - Insurers could take advantage of the low-beta anomaly by selling more insurance policies, issuing public debt, or paying out dividends.
- Insurance pricing.
  - Insurers could increase leverage by selling more insurance policies at lower prices.
- Agency problems (Basak et al. 2007, Huang et al. 2011).
  - Risk-shifting motives could arise from limited liability and state guaranty associations.

Interest risk hedging by insurance and pension sectors

- Negative relation between the 30- minus 10-year government yield spread and the size of the pension and insurance sectors.



Source: Greenwood and Vissing-Jorgensen (2018, Figure 2)

Summary

- Same model that explains insurance pricing, contract design, and reinsurance could also explain portfolio choice.
- Broader question: Quantitative impact of capital requirements on asset pricing and market size in fixed income markets.
- Demand system asset pricing (Koijen and Yogo 2019) could be used to answer these types of questions.