

# Financial Economics of Insurance

## Modeling Demand<sup>1</sup>

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## Motivation

- ▶ Even if the goal is to study supply, need to estimate demand elasticities to decompose prices into markups (market power) versus marginal cost (financial frictions).
- ▶ Need to model insurance market equilibrium for counterfactuals and welfare analysis.
- ▶ Approaches to modeling demand.
  1. Life-cycle model: Cannot explain why demand is different for close substitutes.
  2. Demand systems in product space (e.g., Deaton and Muellbauer 1980).
  3. Demand systems in characteristics space (e.g., Berry, Levinsohn, and Pakes 1995).

## Why model demand in characteristics space?

- ▶ Insurers offer many policies that are close substitutes.
- ▶ These policies are differentiated by contract characteristics (e.g., maturity) and insurer characteristics (e.g., rating).
- ▶ Demand systems in characteristics space naturally model products as close substitutes if they have similar characteristics.
- ▶ Parsimonious model whose parameters increase in the number of characteristics (not number of products).
- ▶ Other applications of random coefficients logit model in retail financial markets.
  - ▶ Health insurance (Einav et al. 2017).
  - ▶ Banking (Egan et al. 2017).
  - ▶ Mutual funds (Hortaçsu and Syverson 2004, Shin 2014).

## Discrete choice problem

- ▶ Customer can choose one of  $I$  contracts or an outside asset.
- ▶ Indirect utility from choosing contract  $i$ :

$$U_{i,t} = -\alpha_P P_{i,t} + \alpha_r r_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t} + \epsilon_{i,t}$$

- ▶  $P_{i,t}$ : Fee.
  - ▶  $r_{i,t}$ : Rollup rate.
  - ▶  $\mathbf{x}_{i,t}$ : Contract and insurer characteristics.
  - ▶  $\xi_{i,t}$ : Unobserved (to the econometrician) characteristics.
  - ▶  $\epsilon_{i,t}$ : Logit error drawn from a type 1 extreme value distribution.
- ▶ Probability that customer chooses contract  $i$  is

$$q_{i,t}(\alpha_P) = \frac{\exp(\alpha_P P_{i,t} + \alpha_r r_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t})}{1 + \sum_{j=1}^I \exp(\alpha_P P_{j,t} + \alpha_r r_{j,t} + \beta' \mathbf{x}_{j,t} + \xi_{j,t})}$$

## Random coefficients logit model

- ▶ Heterogeneity in demand elasticity  $\alpha_P$  across customers, drawn from a lognormal distribution.
- ▶ Market share for contract  $i$  in period  $t$ :

$$Q_{i,t} = \int q_{i,t}(\alpha_P) dF(\alpha_P)$$

- ▶ Cross-price elasticity is

$$\frac{\partial \log(Q_{i,t})}{\partial \log(P_{j,t})} = \frac{P_{j,t}}{Q_{i,t}} \int -\alpha q_{i,t}(\alpha_P) q_{j,t}(\alpha_P) dF(\alpha_P)$$

- ▶ More realistic than the logit model, which implies substitution proportional to market share.

$$\frac{\partial \log(Q_{i,t})}{\partial \log(P_{j,t})} = -\alpha P_{i,t} Q_{j,t}$$

## Estimation of the random coefficients logit model

- ▶ Estimation by GMM based on

$$\mathbb{E}[\xi_{i,t} | \mathbf{z}_{i,t}, \mathbf{x}_{i,t}] = 0$$

- ▶  $\xi_{i,t}$  is not analytical, so evaluation of the moment function requires a simulation.
- ▶ Rewrite the market share as

$$Q(\mu_P, \sigma_P^2, \boldsymbol{\delta}_t) = \int \frac{\exp(-e^{-\nu} P_{i,t} + \delta_{i,t})}{1 + \sum_{j=1}^I \exp(-e^{-\nu} P_{j,t} + \delta_{j,t})} dF(\nu)$$

where

$$\delta_{i,t} = \alpha_r r_{i,t} + \boldsymbol{\beta}' \mathbf{x}_{i,t} + \xi_{i,t}$$

$$\nu \sim \mathbb{N} \left( \mu_P - \frac{\sigma_P^2}{2}, \sigma_P^2 \right)$$

## Evaluation of the moment function

1. Start with initial guess  $(\mu_P, \sigma_P^2, \alpha_r, \beta)$  and  $\delta_t(1)$ .
2. Iterate until convergence:

$$\delta_t(n+1) = \delta_t(n) + \log(\mathbf{Q}_t) - \log(\mathbf{Q}_t(\mu_P, \sigma_P^2, \delta_t(n)))$$

3. Compute  $\xi_{i,t} = \delta_{i,t}(N) - \alpha_r r_{i,t} - \beta' \mathbf{x}_{i,t}$ .

## Application to variable annuities

- ▶ Contract characteristics: Fee, rollup rate, number of investment options, dummy for GLWB, and share class fixed effects.
  - ▶ Share class B: Surrender charge but no sales charge.
  - ▶ Share class I: No commissions for investment advisors.
- ▶ Insurer characteristics: A.M. Best rating and fixed effects.
- ▶ Outside asset: Mutual funds.



## Further work on modeling demand

- ▶ Brokers steer variable annuity customers based on commissions.
  - ▶ Bhattacharya, Illanes, and Padi (2022); Egan, Ge, and Tang (2022); Barbu (2021).
- ▶ Inattentive or not fully rational customers.
  - ▶ French life insurers sell euro contracts that have predictable variation in returns due to reserve dynamics. Customers are inelastic to expected returns (Hombert and Lyonnet 2022).

# Instruments

- ▶ Two endogenous variables (fee and rollup rate) means that we need two instruments.
- 1. Reserve valuation: Value of existing liabilities.
  - ▶ Reserves determined by AG43 (30% conditional tail expectation). No reason to believe policyholders value VA the same way as regulators.
- 2. Reinsurance share of VA.
  - ▶ Constrained insurers use reinsurance.
  - ▶ Most of reinsurance is shadow insurance, which policyholders have little knowledge of beyond what's reflected in ratings.

## Estimated model of variable annuity demand

Variable	Mean	Standard deviation
Fee	3.37 (0.13)	0.30 (0.05)
Rollup rate	0.18 (0.01)	
Investment options	0.11 (0.01)	
GLWB	17.02 (2.64)	
Share class		
A	-9.01 (1.60)	
C	2.01 (0.62)	
I	-13.82 (2.34)	
L	4.99 (1.05)	
O	-5.60 (1.03)	
X	3.86 (0.82)	
A.M. Best rating	0.73 (0.10)	
Observations	32,419	

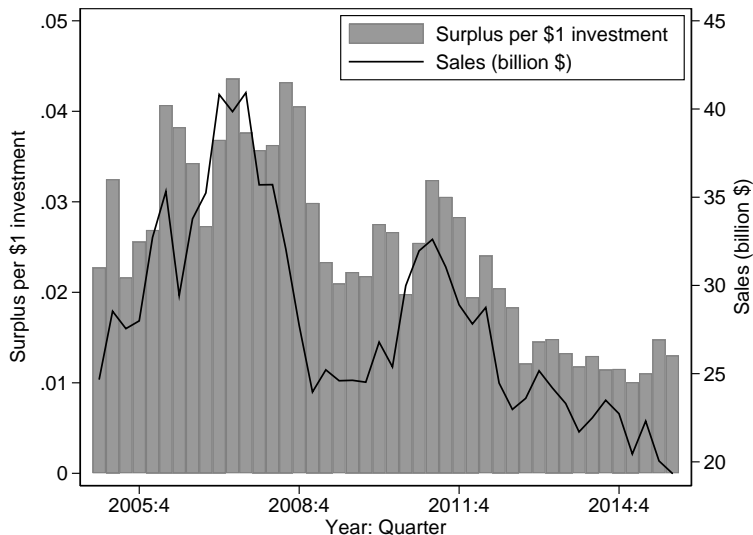
## Consumer surplus

- ▶ What is the total consumer surplus from variable annuities?
- ▶ Log-sum formula (Small and Rosen 1981):

$$\int \frac{\log(1 + \sum_{i=1}^I \exp(-\alpha_P P_{i,t} + \alpha_r r_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t}))}{\alpha_P} dF(\alpha_P)$$

- ▶ Analogous to asking what is the value of defined benefit plans, relative to an alternative in which all plans are defined contribution.
- ▶ Possible because we observe both fees and quantities for variable annuities.

## Consumer surplus from variable annuities



## Decomposition of fees and rollup rates

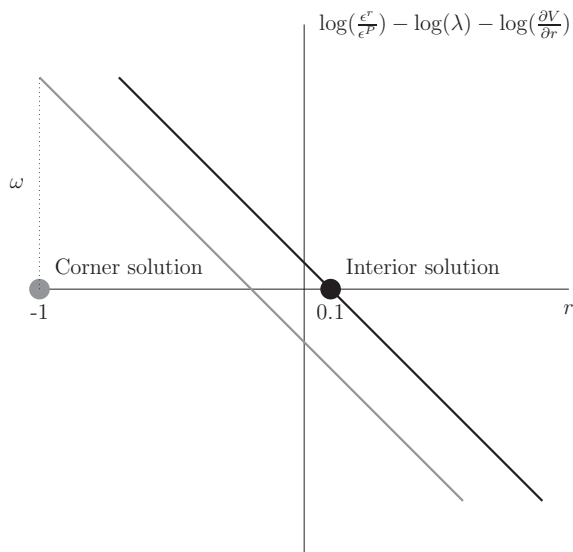
- ▶ Optimality condition for fee:

$$\log \left( P_{i,n,t} - \frac{1}{\epsilon_{i,n,t}^P} \right) = \log(V_{i,n,t}) + \log(\lambda_{n,t})$$

- ▶ Optimality condition for rollup rate:

$$\log \left( \frac{\epsilon_{i,n,t}^r}{\epsilon_{i,n,t}^P} \right) - \log(\lambda_{n,t}) - \log \left( \frac{\partial V_{i,n,t}}{\partial r_{i,n,t}} \right) = \omega_{i,n,t} \geq 0$$

## Interior versus corner solution for the rollup rate



## Parametric assumptions

- ▶ Option value:

$$V_{i,n,t} = \exp(\delta' \mathbf{y}_{i,n,t} + \exp(\mathbf{\Delta}' \mathbf{y}_{i,n,t}) r_{i,n,t} + \nu_{i,n,t})$$

- ▶ Slope of option value:

$$\frac{\partial V_{i,n,t}}{\partial r_{i,n,t}} = \exp(\mathbf{\Delta}' \mathbf{y}_{i,n,t}) V_{i,n,t}$$

- ▶ Shadow cost of capital:

$$\lambda_{n,t} = \exp(\mathbf{\Gamma}' \mathbf{z}_{n,t} + \gamma' \mathbf{1}_n)$$



## Moment conditions

- ▶ Moment conditions for the optimal fee:

$$\mathbb{E} \left[ \nu_{i,n,t} \begin{pmatrix} \mathbf{y}_{i,n,t} \\ \mathbf{y}_{i,n,t} r_{i,n,t} \\ \mathbf{z}_{n,t} \\ \mathbf{1}_n \end{pmatrix} \right] = \mathbf{0}$$

- ▶ Moment inequalities for the optimal rollup rate:

$$\mathbb{E}[\omega_{i,n,t} \mathbf{y}_{i,n,t}] \geq \mathbf{0}$$

- ▶ Transform into moment equalities as

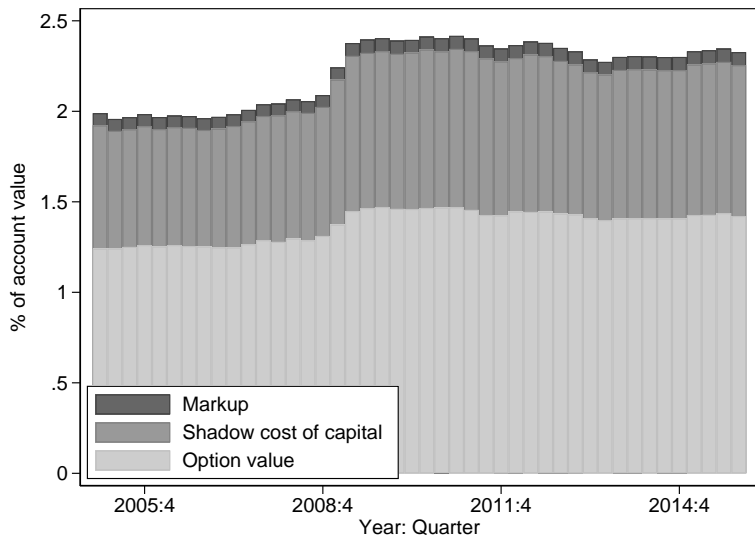
$$\mathbb{E}[\text{diag}(\omega_{i,n,t} \mathbf{1} - \mathbf{\Omega}) \mathbf{y}_{i,n,t}] = \mathbf{0}$$

where  $\mathbf{\Omega} \geq \mathbf{0}$ .

## Estimated model of variable annuity supply

Variable	Coefficient	Standard error
<i>Panel A. Level of option value</i>		
Investment options	0.44	(0.02)
GLWB	46.05	(0.25)
Share class		
A	-44.01	(0.93)
C	11.53	(0.36)
I	-78.83	(1.39)
L	18.78	(0.26)
O	-19.08	(1.13)
X	16.06	(0.33)
<i>Panel B. Slope of option value</i>		
Investment options	2.90	(0.13)
GLWB	-21.67	(0.91)
Share class		
A	29.41	(5.15)
C	5.09	(1.31)
I	89.48	(1.85)
L	-28.94	(2.51)
O	13.92	(10.15)
X	-16.88	(3.45)
<i>Panel C. Shadow cost of capital</i>		
A.M. Best rating	-2.08	(0.25)
Reserve valuation	0.48	(0.25)
Variable annuities reinsured	0.97	(0.20)
<i>Panel D. Constraint on the rollup rate</i>		
Investment options	0.00	
GLWB	0.00	
Share class		
A	26.73	(4.63)

## Decomposition of fees



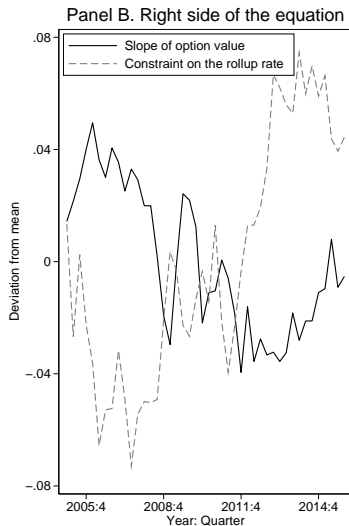
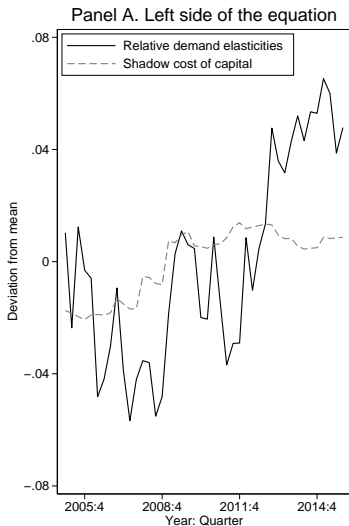
## Decomposition of rollup rates

- ▶ Equation for the optimal rollup rate:

$$\mathbb{E} \left[ \log \left( \frac{\epsilon_{i,n,t}^r}{\epsilon_{i,n,t}^P} \right) - \log(\lambda_{n,t}) \right] = \mathbb{E} \left[ \log \left( \frac{\partial V_{i,n,t}}{\partial r_{i,n,t}} \right) + \omega_{i,n,t} \right]$$

- ▶ After global financial crisis,
  1. Relative demand elasticities increased.
  2. Shadow cost of capital increased.
  3. Slope of option value decreased as insurers reduced rollup rates.
  4. Constraint on the rollup rate binding.

# Decomposition of rollup rates



# Summary

- ▶ An equilibrium framework to study insurance markets.
  - ▶ Supply: Choose prices and contract characteristics to maximize firm value subject to risk-based capital cost.
  - ▶ Demand: Random coefficients logit model.
  - ▶ Demand estimation.
  - ▶ Use the model for policy counterfactuals and welfare analysis.
- ▶ Next application: Reinsurance.