Financial Economics of Insurance
Modeling Supply

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Composition of life insurers’ liabilities in 2017

<table>
<thead>
<tr>
<th>Liability</th>
<th>Trillion $</th>
</tr>
</thead>
<tbody>
<tr>
<td>General account</td>
<td></td>
</tr>
<tr>
<td>Life insurance</td>
<td>1.2</td>
</tr>
<tr>
<td>Annuities</td>
<td>1.2</td>
</tr>
<tr>
<td>Pension funds</td>
<td>0.7</td>
</tr>
<tr>
<td>Other (including accident &amp; health)</td>
<td>0.8</td>
</tr>
<tr>
<td>Separate account (variable annuities)</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Risk-sharing functions of life insurers:

1. Diversify idiosyncratic risk: Traditional life/health products.
2. Market risk insurance: Reallocate aggregate risk across
   - Investors with heterogeneous risk preferences (Dumas 1989).
   - Cohorts, taking on the traditional role of pension plans and Social Security (Allen and Gale 1997).
Questions

1. What do life insurers insure?
   - Market risk through minimum return guarantees.
   - **Variable annuity** = Mutual fund + Long-dated put option
   - $1.5 trillion or 35% of US life insurers’ liabilities in 2015.

2. How do they insure market risk?
   - Financial frictions and market power interact with aggregate shocks.
   - After the global financial crisis,
     - **Pricing**: Fees increased and sales decreased.
     - **Contract characteristics**: Insurers made guarantees less generous or exited to limit risk exposure.
     - Moved liabilities off balance sheet through reinsurance.
   - Never fully recovered because of the low interest rate environment.
     - Negative duration and negative convexity.
     - During 2010–2017, insurance stocks have 10-year Treasury beta of $-1.28$ (0.16).
How are variable annuities regulated?

- Insurance regulators and rating agencies use the risk-based capital ratio.

\[ \text{RBC} = \frac{\text{Assets} - \text{Reserves}}{\text{Required capital}} \]

- Value of minimum return guarantee (put option) increases with lower stock prices, lower interest rates, and higher volatility.
  1. Reserves increase because of revaluation.
  2. Required capital increases through risk exposure.

- Accounting value of minimum return guarantees:

\[ \text{Reserve valuation} = \frac{\text{Reserves}}{\text{Account value}} \]
Value-at-risk constraint

- Insurers may face an economic constraint (in addition to a regulatory constraint).
- Value-at-risk constraint:

\[
\Pr \left( \frac{\text{Liabilities}}{\text{Assets}} \leq 1 \right) = F \left( \frac{\text{Assets}}{\text{Liabilities}} \right) \geq \kappa
\]

- Rewrite as

\[
\frac{\text{Assets} - \text{Liabilities}}{\phi \text{Liabilities}} \geq 1
\]

where \( \phi = F^{-1}(\kappa) - 1 \).

- Insurer with more conservative risk management chooses higher \( \kappa \) or lower variance.
Data on the variable annuity market

  - Quarterly contract-level data on sales, fees, and characteristics.

  - Annual data on VA account value, reserve value, and amount of reserves reinsured.

  - Annual financial statements.
  - A.M. Best rating.
Variable annuity sales

- Variable annuity sales (billion $)
- Mutual fund sales (billion $)
Number of insurers and contracts with minimum return guarantees

![Graph showing the number of insurers and contracts over years and quarters.](image)
Fees on minimum return guarantees

Panel A. Fee

<table>
<thead>
<tr>
<th>Year: Quarter</th>
<th>Total fee</th>
<th>Fee on MRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011:4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014:4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rollup rates on minimum return guarantees

Panel B. Rollup rate

- Share of contracts with MRG
- Rollup rate (%)
Evidence of supply-side fragility

- Fees increased and sales decreased.
- **Changing contract characteristics:** Insurers made guarantees less generous to limit risk exposure.
  - Lower rollup rates (intensive margin).
  - Exit the market for guarantees (extensive margin).
- **Hypothesis:** Higher valuation of existing liabilities lowers risk-based capital.
- In the cross section, insurers with larger increases in reserve valuation
  1. Reduce sales more.
Cross section of insurers during the global financial crisis

Panel A. Sales growth

Change in reserve valuation vs. Sales growth (%)

Offered GLWB in 2007:
- Yes
- No
Cross section of insurers during the global financial crisis

Panel B. Variable annuity reinsurance

Offered GLWB in 2007:
- Yes
- No

Change in reinsurance share (%) vs. Change in reserve valuation
A model of variable annuity supply

▶ Key frictions:

▶ In response to an adverse shock to risk-based capital, insurer
  1. Increases fees.
  2. Changes contract characteristics (or exits entirely) to limit risk exposure.
Variable annuity market

- Financial market:
  - Asset price $S_t$ evolves exogenously.
  - SDF that prices all assets: $S_t = \mathbb{E}_t[M_{t,t+1}S_{t+1}]$.

- VA issued in period $t$ matures in period $t+2$.
  - Fee $P_t$ (per $1$ account value).
  - Rollup rate $r_t$.
  - Payoff of minimum return guarantee:
    \[
    X_{t,t+2} = \max\left\{ (1 + r_t)^2 - \frac{S_{t+2}}{S_t}, 0 \right\}
    \]

  - Option value: $V_{t,t} = \mathbb{E}_t[M_{t,t+2}X_{t,t+2}]$.

- VA is a mutual fund when $r_t = -1$ (i.e., insurer exits the market for guarantees).
Balance sheet dynamics

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_t$</td>
<td>$B_t$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>$L_t$</td>
</tr>
<tr>
<td></td>
<td>$A_t - L_t$</td>
</tr>
</tbody>
</table>

- **Account value:**

$$B_t = \frac{S_t}{S_{t-1}} B_{t-1} + Q_t$$

- **Assets:**

$$A_t = R_{A,t} A_{t-1} + P_t Q_t$$

- **Liabilities:**

$$L_t = \frac{V_{t-1,t}}{V_{t-1,t-1}} L_{t-1} + V_{t,t} Q_t$$
Risk-based capital

- Risk-based capital:
  \[ K_t = A_t - L_t - \phi_t L_t \]
  - equity
  - required capital

- Risk weight \( \phi_t > 0 \) on liabilities.

- Cost of a rating downgrade or regulatory action:
  \[ C_t = C(K_t) \]
  where \( C' < 0 \) and \( C'' > 0 \).

- Define shadow cost of capital as
  \[ \lambda_t = \frac{1 + c_t(1 + \phi_t)}{1 + c_t} > 1 \]
  where \( c_t = -\partial C_t/\partial K_t \).
Shocks to risk-based capital

$V_{t-1,t-1}$ $V_{t-1,t}$ $V_{t,t}$ $V_{t,t+1}$

$t - 1$ $t$ $t$ $t + 1$

- Adverse shock at $t$ (i.e., lower stock prices, lower interest rates, or higher volatility).
  1. Value of existing liabilities $V_{t-1,t}$ rises, lowering risk-based capital.
  2. Marginal cost $V_{t,t}$ increases on new contracts.
- Shadow cost of capital $\lambda_t$ increases through $V_{t-1,t}$. 
Insurer's maximization problem

- **Demand:** $Q_t = Q_t(P_t, r_t)$.
  - Decreasing in fee: $\partial Q_t / \partial P_t < 0$.
  - Increasing in the rollup rate: $\partial Q_t / \partial r_t > 0$.
- **Insurer cannot offer negative rollup rates:** $r_t \in \{-1\} \cup (-0.01, \infty)$.
- **Insurer maximizes firm value:**
  $$\max_{P_t, r_t} (P_t - V_{t,t})Q_t - C_t.$$
Optimal pricing

▶ Optimal fee is

\[ P_t = \frac{1}{\epsilon_t} + \lambda_t V_{t,t} \]

1. Decreases in semi-elasticity \( \epsilon_t \).
2. Increases in option value \( V_{t,t} \).
3. Increases in shadow cost of capital \( \lambda_t \).

▶ Adverse shock increases fee through

1. Higher \( V_{t,t} \).
2. Higher \( \lambda_t \) through revaluation of existing liabilities.
Optimal rollup rate

- At an interior optimum, rollup rate satisfies

\[ \frac{\epsilon^r_t}{\epsilon^P_t} = \lambda_t \frac{\partial V_{t,t}}{\partial r_t} \]

Otherwise, corner solution \( r_t = -1 \).

- When shadow cost of capital \( \lambda_t \) is high, insurer
  - Reduces the rollup rate.
  - Can exit the market for guarantees (i.e., \( r_t = -1 \)).

- **Key insight:** Contract characteristics respond to risk-based capital and can lead to market incompleteness.
# Supply-side theory of insurance

<table>
<thead>
<tr>
<th></th>
<th>Traditional view</th>
<th>Supply-side view</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Products</strong></td>
<td>Life/health insurance &amp; traditional annuities</td>
<td>Minimum return guarantees</td>
</tr>
<tr>
<td><strong>Insures</strong></td>
<td>Idiosyncratic risk across states</td>
<td>Market risk across investors</td>
</tr>
<tr>
<td><strong>Frictions</strong></td>
<td>Informational</td>
<td>Financial/regulatory &amp; market power</td>
</tr>
<tr>
<td><strong>Consequences</strong></td>
<td>Variation in prices, contract characteristics &amp; degree of market incompleteness</td>
<td></td>
</tr>
</tbody>
</table>
Broader issues

- Mutual funds traditionally pure pass-through institutions with no risk mismatch.
- Growing part of the mutual fund sector sold through life insurers is subject to risk mismatch.
- Similar problem to persistent under-funding of pension funds, but with additional market discipline for publicly traded companies.
- Minimum return guarantees are a significant share of life insurers’ liabilities in Austria, Denmark, France, Germany, Netherlands, and Sweden (ESRB 2015).
- A potential issue to monitor for financial stability.