Financial Economics of Insurance
Modeling Supply[1]

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[1]Based upon work supported by NSF grant 1727049.

Composition of life insurers' liabilities in 2017

Liability	Trillion $
General account	
Life insurance	1.2
Annuities	1.2
Pension funds	0.7
Other (including accident & health)	0.8
Separate account (variable annuities)	2.7

Risk-sharing functions of life insurers:

1. Diversify idiosyncratic risk: Traditional life/health products.
2. Market risk insurance: Reallocate aggregate risk across
   - Investors with heterogeneous risk preferences (Dumas 1989).
   - Cohorts, taking on the traditional role of pension plans and Social Security (Allen and Gale 1997).

Questions

1. What do life insurers insure?
   - Market risk through minimum return guarantees.
   - Variable annuity = Mutual fund + Long-dated put option
   - $1.5 trillion or 35% of US life insurers' liabilities in 2015.
2. How do they insure market risk?
   - Financial frictions and market power interact with aggregate shocks.
   - After the global financial crisis,
     - Pricing: Fees increased and sales decreased.
     - Contract characteristics: Insurers made guarantees less generous or exited to limit risk exposure.
     - Moved liabilities off balance sheet through reinsurance.
   - Never fully recovered because of the low interest rate environment.
     - Negative duration and negative convexity.
     - During 2010–2017, insurance stocks have 10-year Treasury beta of -1.28 (0.16).

How are variable annuities regulated?

- Insurance regulators and rating agencies use the risk-based capital ratio.

$$\text{RBC} = \frac{\text{Assets} - \text{Reserves}}{\text{Required capital}}$$

- Value of minimum return guarantee (put option) increases with lower stock prices, lower interest rates, and higher volatility.
  1. Reserves increase because of revaluation.
  2. Required capital increases through risk exposure.
- Accounting value of minimum return guarantees:

$$\text{Reserve valuation} = \frac{\text{Reserves}}{\text{Account value}}$$

Value-at-risk constraint

- Insurers may face an economic constraint (in addition to a regulatory constraint).
- Value-at-risk constraint:

$$\Pr\left(\frac{\text{Liabilities}}{\text{Assets}}\epsilon \leq 1\right) = F\left(\frac{\text{Assets}}{\text{Liabilities}}\right) \geq \kappa$$

- Rewrite as

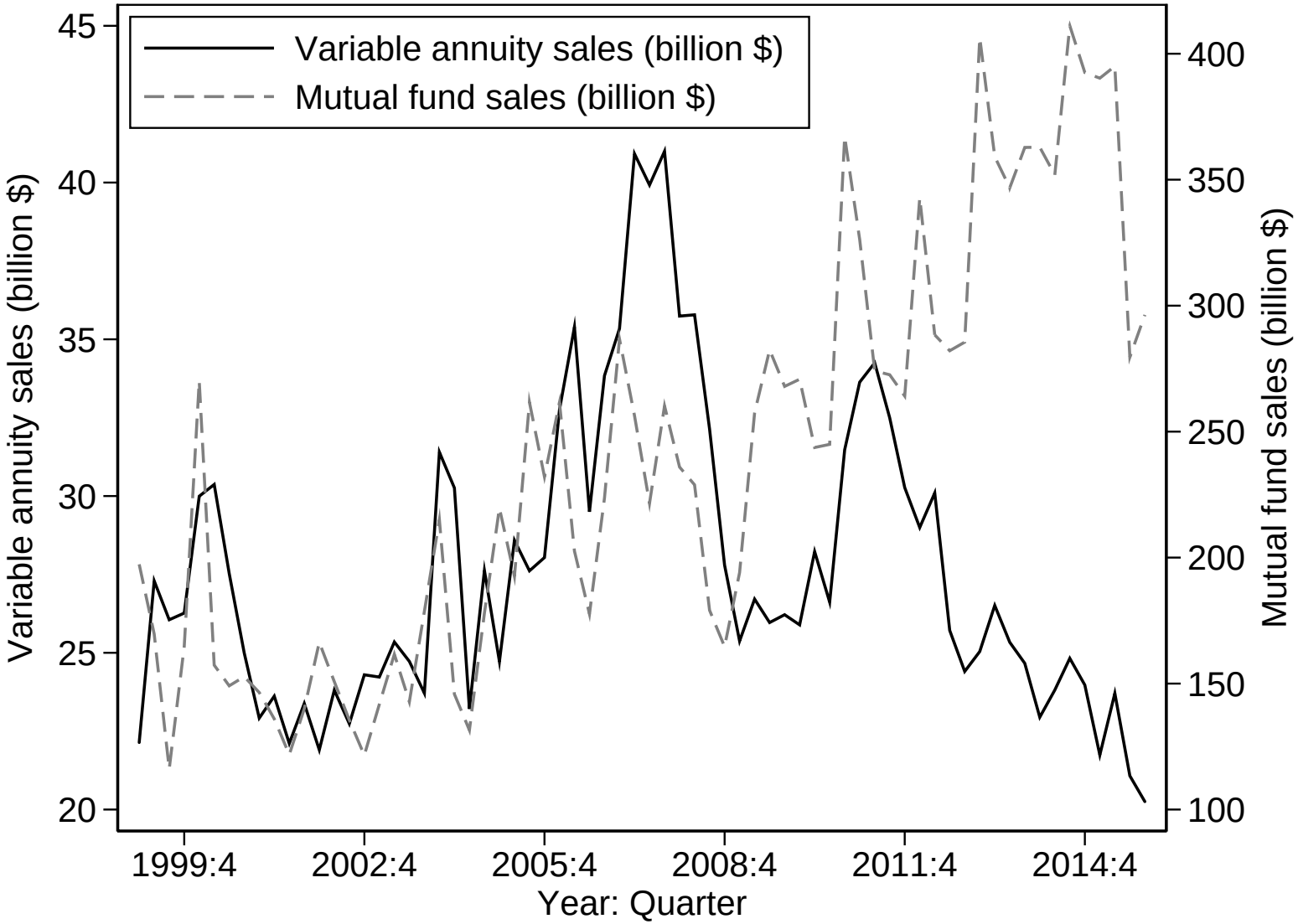
$$\frac{\text{Assets} - \text{Liabilities}}{\phi\text{Liabilities}} \geq 1$$

  where $\phi = F^{-1}(\kappa) - 1$.
- Insurer with more conservative risk management chooses higher κ or lower variance.

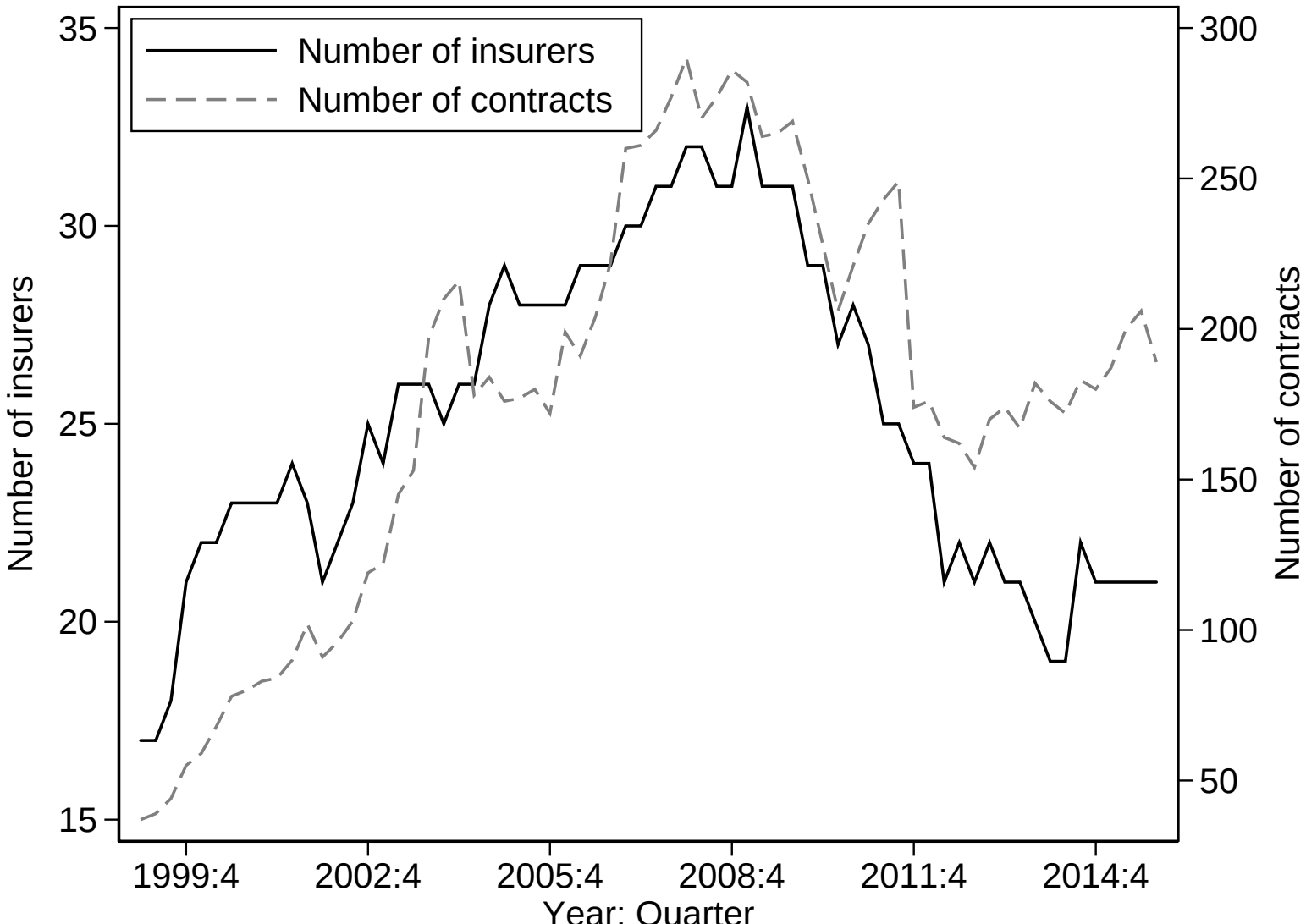
Data on the variable annuity market

- Morningstar (1999:1–2015:4):
  - Quarterly contract-level data on sales, fees, and characteristics.
- NAIC General Interrogatories Part 2 Table 9.2 (2005–2015):
  - Annual data on VA account value, reserve value, and amount of reserves reinsured.
- A.M. Best Company (2005–2015):
  - Annual financial statements.
  - A.M. Best rating.

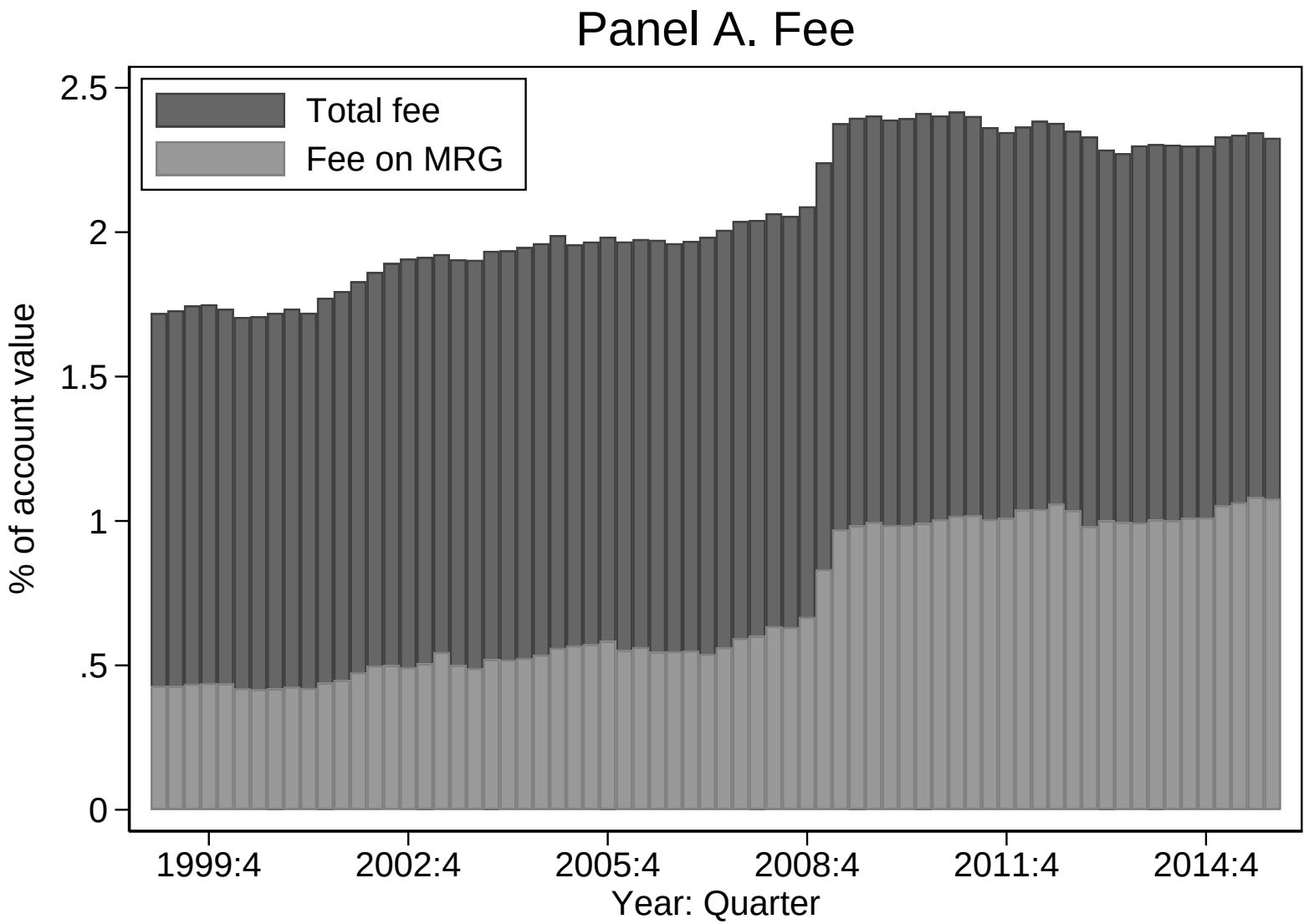
Variable annuity sales



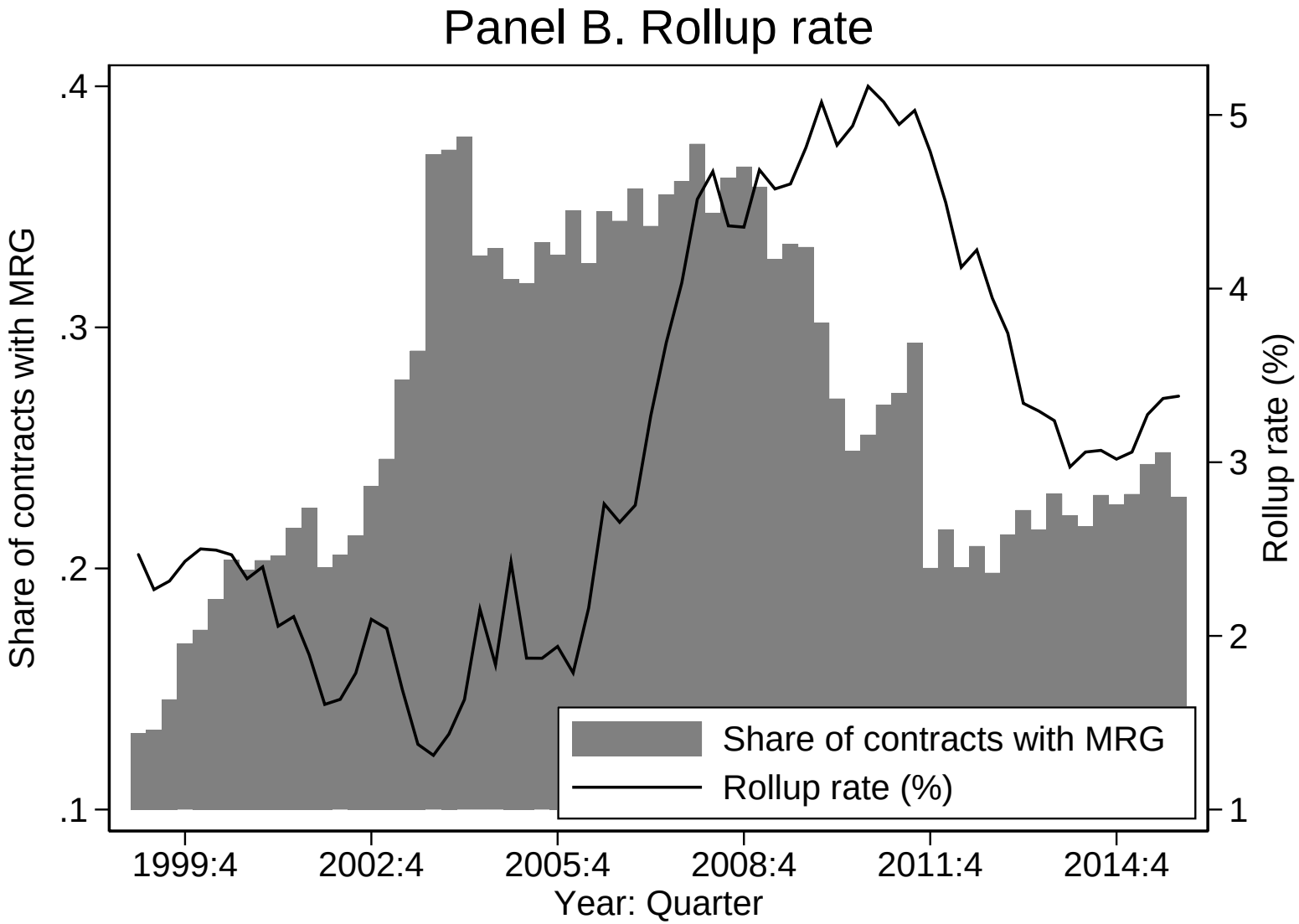
Number of insurers and contracts with minimum return guarantees



Fees on minimum return guarantees



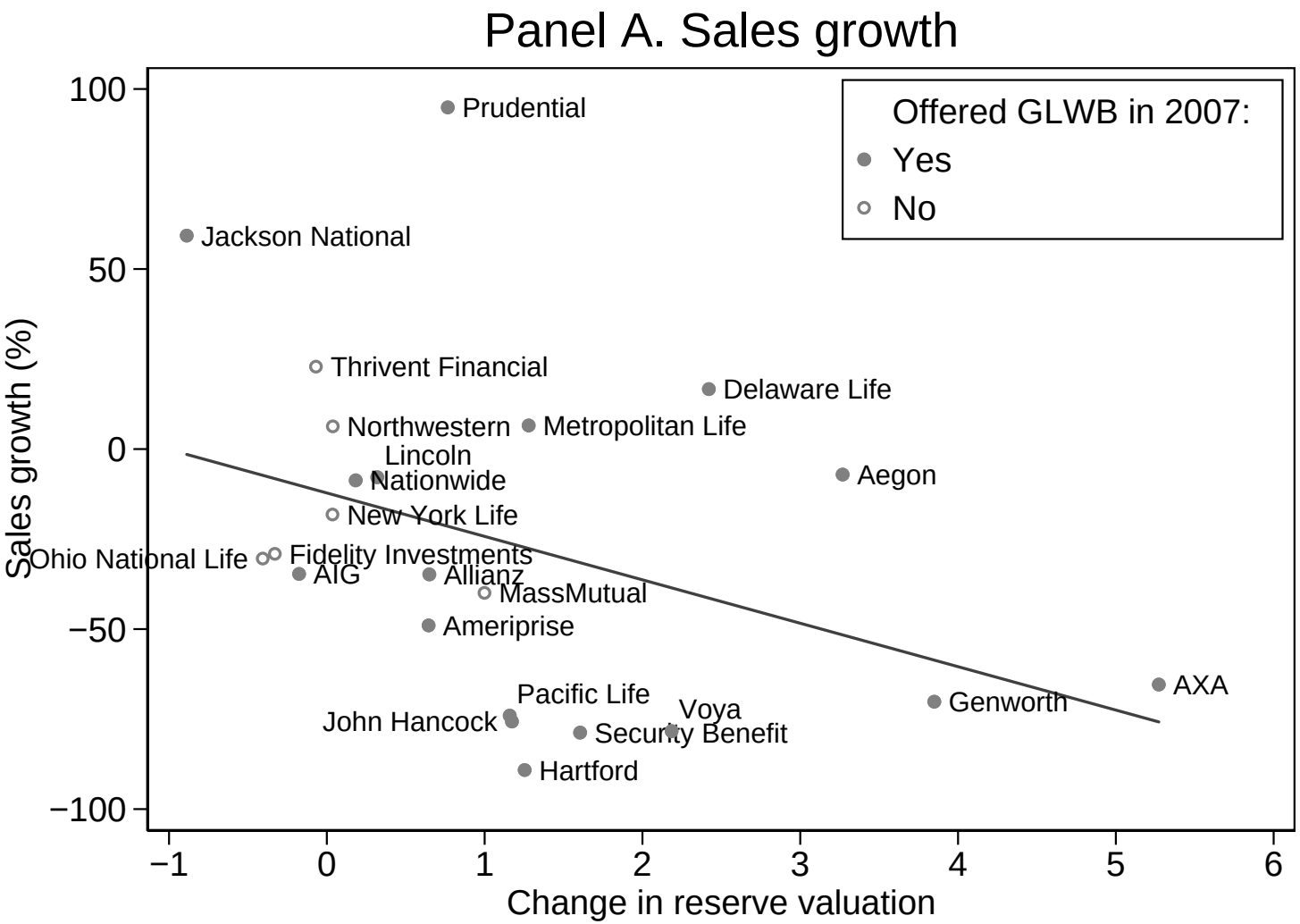
Rollup rates on minimum return guarantees



Evidence of supply-side fragility

- Fees increased and sales decreased.
- Changing contract characteristics: Insurers made guarantees less generous to limit risk exposure.
  - Lower rollup rates (intensive margin).
  - Exit the market for guarantees (extensive margin).
- Hypothesis: Higher valuation of existing liabilities lowers risk-based capital.
- In the cross section, insurers with larger increases in reserve valuation
  1. Reduce sales more.
  2. Move liabilities off balance sheet through reinsurance.

Cross section of insurers during the global financial crisis



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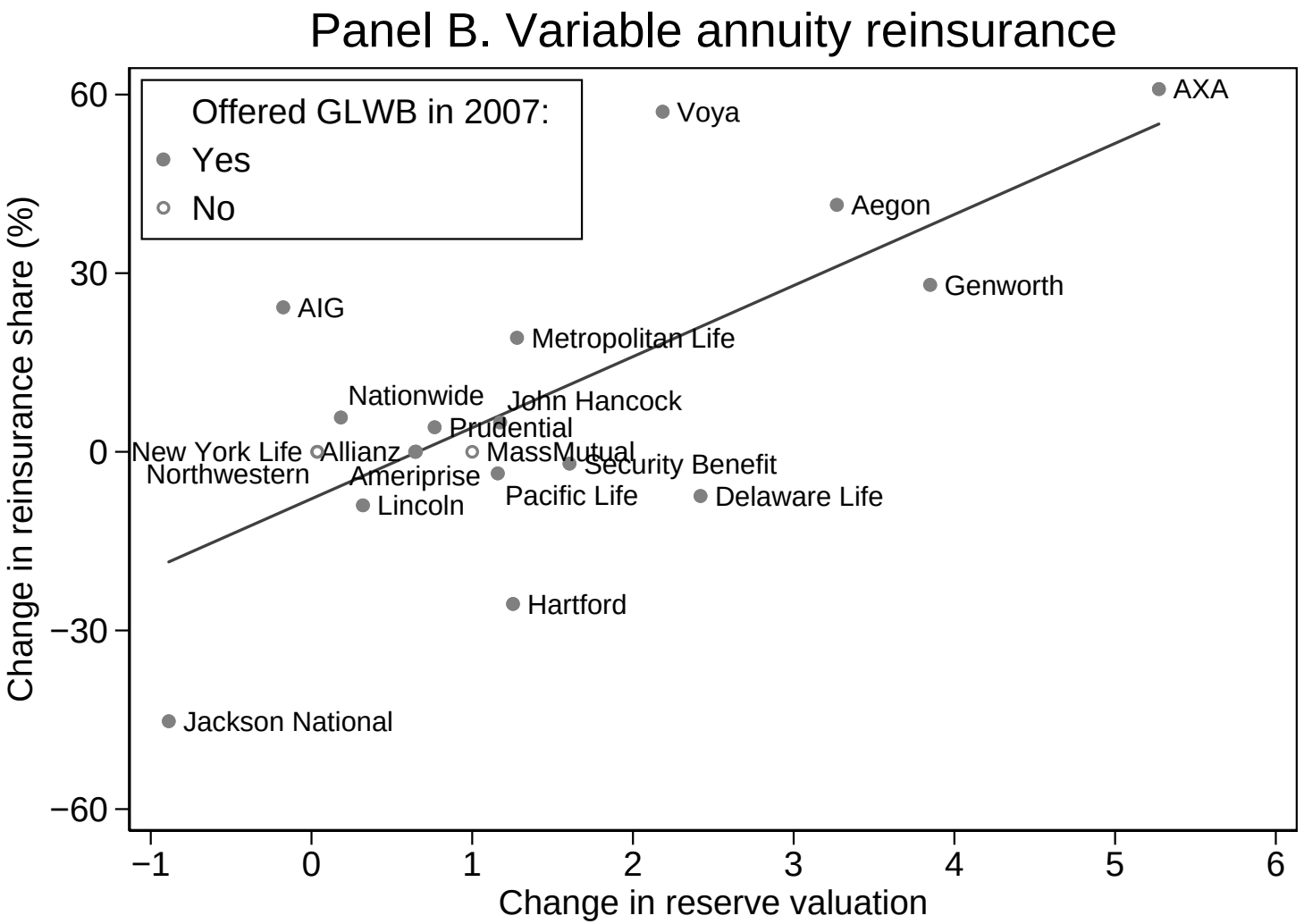


A model of variable annuity supply

- Key frictions:
  1. Financial frictions: Issuance of new contracts increases required capital.
  2. Market power: Insurers compete by Bertrand pricing in an oligopolistic market.
- In response to an adverse shock to risk-based capital, insurer
  1. Increases fees.
  2. Changes contract characteristics (or exits entirely) to limit risk exposure.

Variable annuity market

- Financial market:
  - Asset price S_t evolves exogenously.
  - SDF that prices all assets: $S_t = \mathbb{E}_t[M_{t,t+1} S_{t+1}]$.
- VA issued in period t matures in period $t+2$.
  - Fee P_t (per \$1 account value).
  - Rollup rate r_t.
  - Payoff of minimum return guarantee:

$$X_{t,t+2} = \max\left\{(1+r_t)^2 - \frac{S_{t+2}}{S_t}, 0\right\}$$

  - Option value: $V_{t,t} = \mathbb{E}_t[M_{t,t+2} X_{t,t+2}]$.
- VA is a mutual fund when $r_t = -1$ (i.e., insurer exits the market for guarantees).

Balance sheet dynamics

Assets	Liabilities	
B_t	B_t	separate account
A_t	L_t	general account
	$A_t - L_t$	equity

- Account value:

$$B_t = \frac{S_t}{S_{t-1}} B_{t-1} + Q_t$$

- Assets:

$$A_t = R_{A,t} A_{t-1} + P_t Q_t$$

- Liabilities:

$$L_t = \frac{V_{t-1,t}}{V_{t-1,t-1}} L_{t-1} + V_{t,t} Q_t$$

Risk-based capital

- Risk-based capital:

$$K_t = \underbrace{A_t - L_t}_{\text{equity}} - \underbrace{\phi_t L_t}_{\text{required capital}}$$

  - Risk weight $\phi_t > 0$ on liabilities.
- Cost of a rating downgrade or regulatory action:

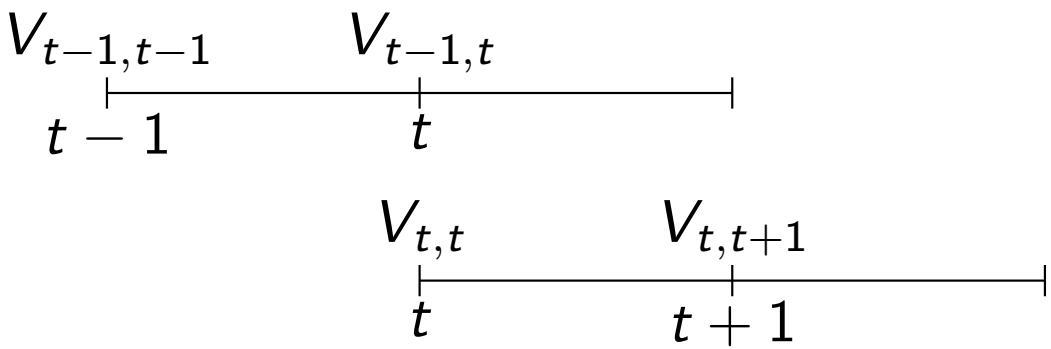
$$C_t = C(K_t)$$

  where $C' < 0$ and $C'' > 0$.
- Define shadow cost of capital as

$$\lambda_t = \frac{1 + c_t(1 + \phi_t)}{1 + c_t} > 1$$

  where $c_t = -\partial C_t / \partial K_t$.

Shocks to risk-based capital



- Adverse shock at t (i.e., lower stock prices, lower interest rates, or higher volatility).
  1. Value of existing liabilities $V_{t-1,t}$ rises, lowering risk-based capital.
  2. Marginal cost $V_{t,t}$ increases on new contracts.
- Shadow cost of capital λ_t increases through $V_{t-1,t}$.

Insurer's maximization problem

- Demand: $Q_t = Q_t(P_t, r_t)$.
  - Decreasing in fee: $\partial Q_t / \partial P_t < 0$.
  - Increasing in the rollup rate: $\partial Q_t / \partial r_t > 0$.
- Insurer cannot offer negative rollup rates: $r_t \in \{-1\} \bigcup (-0.01, \infty)$.
- Insurer maximizes firm value:

$$\max_{P_t, r_t} (P_t - V_{t,t}) Q_t - C_t.$$

Optimal pricing

- Optimal fee is

$$P_t = \frac{1}{\epsilon_t^P} + \underbrace{\lambda_t V_{t,t}}_{\text{marginal cost}}$$

  1. Decreases in semi-elasticity ϵ_t^P.
  2. Increases in option value $V_{t,t}$.
  3. Increases in shadow cost of capital λ_t.

- Adverse shock increases fee through
  1. Higher $V_{t,t}$.
  2. Higher λ_t through revaluation of existing liabilities.

Optimal rollup rate

- At an interior optimum, rollup rate satisfies

$$\frac{\epsilon_t^r}{\epsilon_t^P} = \lambda_t \frac{\partial V_{t,t}}{\partial r_t}$$

  Otherwise, corner solution $r_t = -1$.
- When shadow cost of capital λ_t is high, insurer
  - Reduces the rollup rate.
  - Can exit the market for guarantees (i.e., $r_t = -1$).
- Key insight: Contract characteristics respond to risk-based capital and can lead to market incompleteness.

Supply-side theory of insurance

	Traditional view	Supply-side view
Products	Life/health insurance & traditional annuities	Minimum return guarantees
Insures	Idiosyncratic risk across states	Market risk across investors
Frictions	Informational	Financial/regulatory & market power
Consequences	Variation in prices, contract characteristics & degree of market incompleteness	

Broader issues

- Mutual funds traditionally pure pass-through institutions with no risk mismatch.
- Growing part of the mutual fund sector sold through life insurers is subject to risk mismatch.
- Similar problem to persistent under-funding of pension funds, but with additional market discipline for publicly traded companies.
- Minimum return guarantees are a significant share of life insurers' liabilities in Austria, Denmark, France, Germany, Netherlands, and Sweden (ESRB 2015).
- A potential issue to monitor for financial stability.